Age-Optimal Scheduling for Heterogeneous Traffic With Timely Throughput Constraints

Jingzhou Sun, Lehan Wang, Zhiyuan Jiang, *Member, IEEE*, Sheng Zhou, *Member, IEEE*, and Zhisheng Niu, *Fellow, IEEE*

***Abstract*— We consider a base station supporting two types of traffics, i.e., status update traffic and timely throughput traffic. The goal is to improve the information freshness of status update traffic while satisfying timely throughput constraints. Age of Information (AoI) is adopted as a metric for information freshness. We first propose an age-aware policy that makes scheduling decisions based on the current value of AoI directly. Given timely throughput constraint, an upper bound of the weighted average AoI under this policy is provided. To evaluate policy performance, it is important to obtain the minimum weighted average AoI achievable given timely throughput constraint. A low complexity method is proposed to estimate a lower bound of this value. Furthermore, inspired by the estimation procedure, we design an age-oblivious policy that does not rely on the current AoI to make scheduling decisions. Surprisingly, simulation results show that the weighted average AoI of the age-oblivious policy is comparable to that of the age-aware policy, and both are close to the lower bound.**

***Index Terms*— Age of information, timely throughput, real-time communications, wireless networks.**

I. Introduction

CURRENT communication networks have been designed to maximize throughput. However, with the advent of 5G technologies, more and more efforts have been devoted to developing low latency communication technologies for a wide range of time-sensitive applications [1]. Typical examples include autonomous driving, remote surgery, industrial Internet of Things, where fresh status information plays a critical role.

In view of this, *Age of Information* (AoI) has been proposed in [2] to characterize the information freshness. For a source-destination pair, let $g(t)$ be the generation time of the newest packet received by the destination at time t, the AoI at time t is defined as,

$$\text{AoI}(t) \triangleq t - g(t).$$

This definition implies that a packet containing fresher information can bring in more AoI reduction. Therefore, once such a packet is received, packets with older information are rendered useless. This property especially caters to machine type communication where status update traffic dominates. The control decisions of machines only rely on the freshest information available,[1] which matches the definition of AoI perfectly. Therefore, it is appropriate to adopt AoI as a metric for status update traffic. However, the definition of AoI suggests that AoI is a *process level* metric instead of a *packet level* metric like per-packet delay. The *process level* means that AoI represents how fresh our knowledge is about the status of a process in the sender. When there is no such process, AoI is unsuitable. For example, in vehicular network, it is hard to define a process for applications like emergency alerts, e.g., punctured tire, crashing cars. Traffic like this should be treated at the packet level, and each packet should be delivered within strict deadline, which we call *timely throughput* traffic.

In this work, we consider scheduling problem in wireless network which supports status update traffic and timely throughput traffic simultaneously. AoI is adopted as the metric for the freshness of status update traffic. **Our goal is to design scheduling policies to minimize the weighted average AoI subject to timely throughput constraint under time-varying unreliable channel**. A key problem is how to allocate communication resources to different traffics under distinct channel conditions. The high dimension nature of this problem makes standard approaches like dynamic programming impractical.

As a starting point, we first propose an age-aware scheduling policy based on the Lyapunov optimization method, prove that this policy satisfies the timely throughput constraints, and provide an upper bound of the weighted average AoI under this policy. To evaluate the performance of this policy, we need to compare it with the minimum achievable average AoI, which requires prohibitive computation to obtain. Therefore, we try to compute a lower bound of the minimum average AoI as a substitute, which can be obtained by solving a convex optimization problem. However, since the channel's condition is time-varying, the constraint set of this convex problem is complex. Thus, it is challenging to solve it directly. Based on stochastic approximation, we propose a low complexity method to estimate the optimal value of the convex problem

Manuscript received July 8, 2020; revised December 15, 2020; accepted February 13, 2021. Date of publication March 11, 2021; date of current version April 16, 2021. This work was supported in part by the National Key Research and Development Program of China under Grant 2018YFB0105000; in part by the Nature Science Foundation of China under Grant 61871254, Grant 62022049, Grant 91638204, and Grant 61861136003; and in part by Hitachi Ltd. *(Corresponding author: Sheng Zhou.)*

Jingzhou Sun, Lehan Wang, Sheng Zhou, and Zhisheng Niu are with the Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing 100084, China (e-mail: sunjz18@mails; wang-lh19@mails; sheng.zhou@tsinghua.edu.cn; niuzhs@tsinghua.edu.cn).

Zhiyuan Jiang is with the Shanghai Institute for Advanced Communication and Data Science, Shanghai University, Shanghai 200444, China (e-mail: jiangzhiyuan@shu.edu.cn).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/JSAC.2021.3065059.

Digital Object Identifier 10.1109/JSAC.2021.3065059

[1]Note that we focus on making the real-time decision here. If the goal is to collect data to train a regression model, for example, the assumption that old packets are useless needs to be modified.

online. Furthermore, inspired by this method, we develop an age-oblivious scheduling policy that uses the throughput of status update traffic instead of the current AoI value to make scheduling decisions. It is interesting to find that the performance of this policy is comparable to that of the age-aware scheduling policy, and both are close to the lower bound.

A. Related Work

Since its introduction, AoI has been receiving growing attention for its potential in time-sensitive applications. With various arrival distributions, service distributions, and queueing disciplines, AoI performance is analyzed by utilizing queue theory [3]–[5]. Joint sampling and scheduling problems are studied in [6], [7]. The coding scheme to achieve low AoI is studied in [8].

In this work, we are particularly interested in the scheduling problems for AoI optimization in multiuser scenarios. For centralized scheduling, Kadota *et al.* [9] consider the scheduling problem with deterministic arrivals and develop four scheduling policies. Hsu *et al.* [10] extend the model to allow stochastic arrivals and designs a scheduling policy for zero-buffer scenario based on Whittle's index. It is further improved in [11] to allow arbitrary buffer management schemes. A recent work proves that Whittle's index based scheduling is asymptotically optimal as the number of users grows to infinity [12]. Tang *et al.* [13] use layered decomposition method to design scheduling policy under transmission energy constraints.

In a distributed environment, the AoI minimization problem becomes more challenging due to the lack of a central controller. In [14], the authors study the distributed access problem where the users always have new packets to transmit. They consider the scheme where each user accesses the network with a certain probability, and express the average AoI as a function of the access probability. Jiang *et al.* [15] show that a special round-robin policy with one buffer is asymptotically optimal as the number of users goes to infinity when the channel is reliable. In [16], the authors derive closed-form time average AoI with periodic arrivals and unreliable channels, for round-robin policy and ALOHA random access respectively. For CSMA, in [17], a closed-form average AoI is obtained when the users generate packet at will, and a tight upper bound is obtained when the arrival is stochastic. Furthermore, a contention-based random access scheme with near-optimal performance is proposed in [11]

Timely throughput is first proposed in [18] and is defined as the throughput of packets that are successfully delivered before their deadlines. Two scheduling policies proposed in [18] are shown to satisfy any feasible timely throughput constraint. In [19], this model is generalized to allow unreliable channels and simultaneous arrivals. Extensions include fair resource allocation [20], scheduling with arbitrary arrival pattern and heterogeneous delay constraint [21], etc.

The most relevant to our work are [22]–[25]. In [22], [23], there is only one kind of traffic. In [22], the authors consider the problem of scheduling real-time traffic to minimize the weighted average AoI and satisfy timely throughput constraint of the same traffic. In [23], the authors study the scheduling problem in a network where many users transmit packets to a base station. Their objective is to minimize the average AoI and satisfy the throughput constraint of each user. In [24], [25], the authors consider heterogeneous traffics. In [24], the authors investigate a scenario where one high priority AoI-oriented user competes with N throughput-oriented users. The AoI performance of ALOHA protocol is analyzed. In [25], there are only two users, one is AoI-oriented and the other is delay-oriented. Network calculus is applied to study the trade-off between the delay violation probability and AoI.

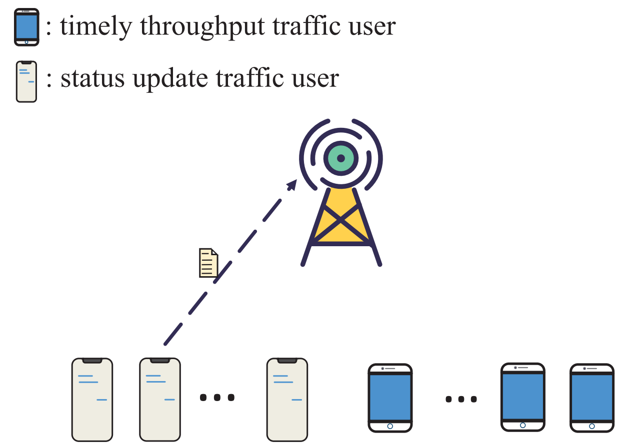

Fig. 1. System Model: A base station supporting status update traffic and timely throughput traffic.

The rest of this paper is organized as follows. In Section II, we describe the system model, characterize the achievable region of timely throughput, and present the optimization problem. In Section III, we design an age-aware scheduling policy based on the Lyapunov optimization method. In Section IV, the problem of computing an AoI lower bound is modeled as a convex optimization problem and we design an iterative method for it. In Section V, we provide an age-oblivious scheduling policy and design an online method to estimate the AoI lower bound. Section VI presents simulation results and discussions. Finally, Section VII concludes the work.

II. System Model

We consider a base station (BS) supporting two kinds of traffics: status update traffic and timely throughput traffic, as illustrated in Fig. 1. The set of users generating status traffic is denoted by $\mathcal{N}$, and its number is $|\mathcal{N}| = N$. $\mathcal{M}$ represents the set of users generating timely throughput traffic, and $|\mathcal{M}| = M$.

Time is slotted and T consecutive slots are grouped into a *frame*. In each time slot, the BS can schedule *one user* to transmit packet. We assume that the scheduled user transmits at most one packet in a slot, either status update traffic or timely throughput traffic. The *transmission decision* in the t-th slot of the $(k+1)$-th frame is $\boldsymbol{D}(kT+t) \triangleq (D_i(kT+t)|i \in \mathcal{M} \cup \mathcal{N})$, where $D_i(kT+t) \in \{0,1\}$, and $D_i(kT+t) = 1$ if user i is scheduled.

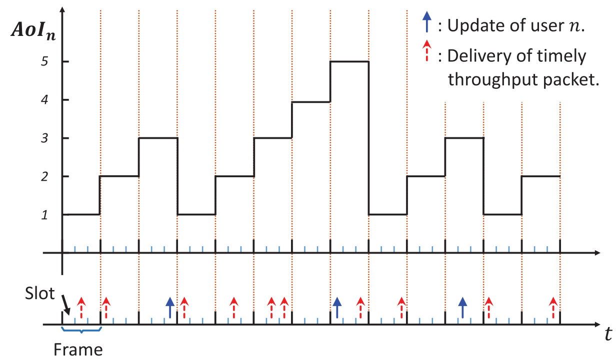

Fig. 2. AoI evolution of user n. The length of a frame is 3 time slots.

We assume the channel is time-varying and discretize the channel state into a finite set $\mathcal{C}$. We also assume that the length of a frame is shorter than the coherence time, and thus the channel state remains *constant* within a frame. Because of the uncertain nature of wireless channel, the transmission may fail. We model this event by Bernoulli random variables: when the channel state is $\boldsymbol{c} \in \mathcal{C}$, the transmission success probability for user i is $p_i(\boldsymbol{c})$. If user i is scheduled and the transmission succeeds, user i will receive an ACK at the end of the slot. *Transmission result* is denoted by vector $\boldsymbol{U}(kT+t) \triangleq (U_i(kT+t)|i \in \mathcal{M} \cup \mathcal{N})$, where $U_i(kT+t) \in \{0,1\}$. $U_i(kT+t) = 1$ if the user i's transmission succeeds.

For users generating status update traffic, we assume that they can sample the underlying process and generate new status packets once scheduled, which is called *active source* [26]. *Age of Information* (AoI) is adopted to measure the freshness of the status packet. Like [9], [22], we assume that the unit of AoI is *frame* instead of *time slot*, as illustrated in Fig. 2. Let $h_n(k)$ be the AoI of user $n \in \mathcal{N}$ at the beginning of the $(k+1)$-th frame. If user n updates in the current frame, h_n decreases to 1 at the beginning of the next frame. Otherwise h_n increases by one, i.e.,

$$h_n(k+1) = h_n(k)\left(1 - \mathbb{I}\left(\sum_{t=0}^{T-1} U_n(kT+t) > 0\right)\right) + 1, \tag{1}$$

where $\mathbb{I}(x)$ is an indicator function. Let $\boldsymbol{H}(k) = (h_n(k)|n \in \mathcal{N})$ be the *AoI vector* in the $(k+1)$-th frame.

As for users generating timely throughput traffic, we assume that traffic arrivals happen at the beginning of a frame as in [18].[2] The *arrival vector* in the $(k+1)$-th frame is $\boldsymbol{a}(k) \triangleq (a_m(k)|m \in \mathcal{M})$, where $a_m(k)$ is the number of arriving packets for user m. We assume that $a_m(k)$ has finite support and $\mathbb{E}[a_m(k)] = \lambda_m$. By the end of each frame, packets that have not been delivered will expire and be dropped. Since the channel is unreliable and the communication resource is limited, it is unavoidable to drop some packets. The maximum allowable drop rate of user m is denoted by ξ_m. The number of delivered packets for user $m \in \mathcal{M}$ in the $(k+1)$-th frame is $d_m(k)$, i.e.,

$$d_m(k) = \min\left(\sum_{t=0}^{T-1} U_m(kT+t), a_m(k)\right). \tag{2}$$

The *timely throughput* of user m is

$$\mu_{t,m} \triangleq \liminf_{K\to\infty} \frac{1}{K}\mathbb{E}\left[\sum_{k=0}^{K-1} d_m(k)\right]. \tag{3}$$

In this work, we assume that the channel state $\boldsymbol{c}(k)$ and arrival vector $\boldsymbol{a}(k)$ are i.i.d between frames. Note that the analysis in this paper can be extended to the case where both $\boldsymbol{c}(k)$ and $\boldsymbol{a}(k)$ are Markovian.

A. Achievable Region

Let $\boldsymbol{\mu} \triangleq (\mu_{t,m}|m \in \mathcal{M})$ be the *timely throughput vector*. We first characterize the achievable region of $\boldsymbol{\mu}$.

Definition 1 (Achievable Region): Timely throughput vector $\boldsymbol{\mu}$ is *achievable* if there exists a scheduling policy $\pi \in \Pi$ whose timely throughput vector $\boldsymbol{\mu}(\pi)$ is no less than $\boldsymbol{\mu}$ on each component. The achievable region Λ is the set of all achievable $\boldsymbol{\mu}$.

The policy set Π consists of all policies under which the BS schedules at most one user in a time slot. A special kind is the *stationary randomized policy* (SR policy). By *stationary*, we mean that this kind of policies are invariant among frames. A stationary randomized policy $\pi \in \Pi^{SR} \subset \Pi$ maps arrival vector $\boldsymbol{a}$ and channel state $\boldsymbol{c}$ to a sequence of conditional probability distributions,

$$\pi : (\boldsymbol{a}, \boldsymbol{c}) \to \{P(\boldsymbol{D}(t)\,|\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{U}(0), \cdots, \boldsymbol{U}(t-1))\}, \tag{4}$$

where $t \in \{0, \cdots, T-1\}$. We omit the counter k for frame because of the stationary property. Note that a SR policy makes decision sequentially, which means that the decision in the t-th time slot relies on the transmission results of the previous $t-1$ slots.

Since the problem of scheduling timely throughput users subject to constraints can be naturally modeled as a Constrained Markov Decision Problem (CMDP), the timely throughput vector $\boldsymbol{\mu}$ that is achieved by policy π can also be achieved by a stationary randomized policy π' [27]. Thus, all timely throughput vectors in the achievable region Λ can be achieved by stationary randomized policies. On the other hand, based on the definition of Λ, all vectors achieved by stationary randomized policies belong to Λ. Therefore, an equivalent definition of the achievable region can be obtained.

Lemma 1: A timely throughput vector $\boldsymbol{\mu} \in \Lambda$ if and only if there exists $\pi \in \Pi^{SR}$ such that

$$\mu_{t,m} \le \sum_{\boldsymbol{a},\boldsymbol{c}} P(\boldsymbol{a})P(\boldsymbol{c})\mathbb{E}_{\pi(\boldsymbol{a},\boldsymbol{c})} \times \left[\min\left(\sum_{t=0}^{T-1} U_m(t), a_m\right)\middle|\, \boldsymbol{a}, \boldsymbol{c}\right]. \tag{5}$$

A direct result is that Λ is a convex, compact set. Compactness is obvious from (5). To see convexity, consider $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2 \in \Lambda$, which can be achieved by π_1 and π_2 respectively. Since the channel state $\boldsymbol{c}(k)$ and arrival vector $\boldsymbol{a}(k)$ are

[2]Timely throughput scheduling with general arrival pattern is very challenging. We start from this modest model to gain some insights.

i.i.d between frames, we can randomly select among π_1 and π_2 at the beginning of each frame. Therefore all convex combinations of $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ can be achieved.

B. Problem Formulation

We now introduce the optimization problem formally:

AoI Minimization with Timely Throughput Constraint

$$\min_{\pi \in \Pi} J(\pi) = \limsup_{K \to \infty} \frac{1}{KN} \mathbb{E}_\pi \left[\sum_{k=0}^{K-1} \sum_{n \in \mathcal{N}} \omega_n h_n(k) \right],$$
$$\text{s.t.} \mu_{t,m}(\pi) \geq \lambda_m (1 - \xi_m), \ \forall m \in \mathcal{M}. \quad (6)$$

ω_n is a predefined weight of user n's AoI, and $\mu_{t,m}(\pi)$ is the timely throughput of user m with policy π. ξ_m represents the strictness of timely throughput constraint. Without loss of generality, we assume that $\lambda_m(1-\xi_m) > 0$. Let π^{Opt} be one of the optimal policies for this problem, and the minimum weighted average AoI is $J(\pi^{\text{Opt}})$.

Whereas this problem can be cast as a CMDP, the prohibitive computation cost of standard iterative method urges us to find low complexity scheduling policies that still enjoy a good performance.

III. Age-Aware Policy

In this section, we first present a low complexity scheduling policy to problem (6). Then, we prove that this policy satisfies all timely throughput constraints in the interior of the achievable region Λ. An upper bound of the weighted average AoI under this policy is provided in the end.

Let Q_m be the virtual queue for user $m \in \mathcal{M}$. Q_m updates as

$$Q_m(k+1) = \max(Q_m(k) + \lambda_m(1-\xi_m) - d_m(k), 0), \quad (7)$$

where $\lambda_m(1-\xi_m)$ is the average number of packets user m has to deliver in a frame to meet the timely throughput constraint, and $d_m(k)$ is the number of packets user m has delivered in the $(k+1)$-th frame. Thus, the virtual queue can be regarded as the difference between the required timely throughput and the actual timely throughput. If the virtual queue is strongly stable, the timely throughput constraint is satisfied [28, Chapter 4], i.e.,

$$\limsup_{K \to \infty} \frac{1}{K} \mathbb{E} \left[\sum_{k=0}^{K-1} Q_m(k) \right] < +\infty \Rightarrow \mu_{t,m} \geq \lambda_m(1-\xi_m). \quad (8)$$

Therefore, the long-term average constraint is turned into a queue stability requirement.

The policy *Age-Aware Policy* (AAP) is presented in Alg.1 where V is an adjustable parameter. In each time slot, AAP first finds the user with the largest $V\omega_n h_n(k) p_n(c)$ value among the status update users that have not delivered yet, and the one with the largest $p_m(c)Q_m(k)$ value among the timely throughput users that have nonempty buffer. The next step is to compare these two values and schedule the user who has the largest weighted AoI or virtual queue. Based on the transmission result, we update the AoI or virtual queue as in Alg.1.[3] Since this policy makes use of the current AoI value, we call it *age-aware*. Although in this work we assume that the BS schedules one user to transmit in each time slot, this is not a restrictive assumption. If there are k subchannels, we can modify Alg.1 by scheduling users with the top k largest values.

Algorithm 1 Age-Aware Policy (AAP)

Require: Parameter V.
$h_n(-1) = 0, \forall n \in \mathcal{N}$; $Q_m(-1) = 0, \forall m \in \mathcal{M}$; $k = -1$.
while 1 **do**
  $k \leftarrow k + 1$
  $h_n(k) \leftarrow h_n(k-1) + 1, \forall n \in \mathcal{N}$
  $Q_m(k) \leftarrow Q_m(k-1) + \lambda_m(1-\xi_m), \forall m \in \mathcal{M}$
  for each slot in the k-th frame **do**
    $n^* \leftarrow \arg\max_{n \in \mathcal{N}} \omega_n h_n(k) p_n(c)$
    $m^* \leftarrow \arg\max_{m \in \mathcal{M},\ a_m(k) > 0} p_m(c) Q_m(k)$
    if $m^* =$ NULL **then**
      Schedule user n^*
      if Transmission succeeds **then**
        $h_{n^*}(k) \leftarrow 0$
      end if
    else if $V\omega_{n^*} h_{n^*}(k) p_{n^*}(c) > p_{m^*}(c) Q_{m^*}(k)$ **then**
      Schedule user n^*
      if Transmission succeeds **then**
        $h_{n^*}(k) \leftarrow 0$
      end if
    else
      Schedule user m^*
      if Transmission succeeds **then**
        $Q_{m^*}(k) \leftarrow \max(Q_{m^*}(k) - 1, 0)$, $a_{m^*}(k) \leftarrow a_{m^*}(k) - 1$
      end if
    end if
  end for
end while

When V is large, AAP is more sensitive to the change in age value, and when V is small, AAP is more inclined to service timely throughput traffic. Nevertheless, the scheduling priority increases with AoI even when V is small. By adjusting V, we can change the distribution of AoI as the simulation results show. Intuitively, such property can prevent AoI and virtual queue from diverging to infinity, and thus (8) is satisfied. To prove this statement we first introduce Lyapunov function $L(k)$ as

$$L(k) \triangleq \frac{1}{2} \sum_{m \in \mathcal{M}} Q_m^2(k). \quad (9)$$

Lyapunov function $L(k)$ has the following property. For ease of presentation, let $\boldsymbol{Q}(k) \triangleq (Q_m(k) | m \in \mathcal{M})$.

[3]For ease of presentation, in this part, AoI drops to 0 once the user has delivered in the current frame. But we still calculate AoI by frame, as mentioned in Section II.

Theorem 1: If there exists $\epsilon > 0$ such that the timely throughput vector $(\lambda_m(1-\xi_m)(1+2\epsilon)|m \in \mathcal{M}) \in \Lambda$, the AoI and virtual queue under AAP satisfy

$$\mathbb{E}\left[L(k+1)-L(k)+V\sum_{n\in\mathcal{N}}\omega_n h_n(k+1)\middle|\boldsymbol{Q}(k),\boldsymbol{H}(k)\right] < B+V\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1)-\frac{WV}{N}\sum_{n\in\mathcal{N}}\omega_n h_n(k) -\epsilon\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k), \quad (10)$$

where

$$W \triangleq \min_{n\in\mathcal{N}}\mathbb{E}_{\boldsymbol{c}}\left[p_n(\boldsymbol{c})\frac{\epsilon}{(1+2\epsilon)}\right],$$
$$B \triangleq \frac{1}{2}\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+T^2.$$

Proof: See Appendix A. ■

Based on Theorem 1, we have the following corollary,

Corollary 1: If there exists $\epsilon > 0$ such that the timely throughput vector $\{\lambda_m(1-\xi_m)(1+2\epsilon)\} \in \Lambda$, then the AAP satisfies the timely throughput constraints $\{\lambda_m(1-\xi_m)\}$ in problem (6).

Proof: Taking expectation on both sides of (10) with respect to $\boldsymbol{Q}(k),\boldsymbol{H}(k)$ yields

$$\mathbb{E}\left[L(k+1)-L(k)+V\sum_{n\in\mathcal{N}}\omega_n h_n(k+1)\right] < B+V\mathbb{E}\left[\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1)\right]-\frac{WV}{N}\mathbb{E}\left[\sum_{n\in\mathcal{N}}\omega_n h_n(k)\right] -\epsilon\mathbb{E}\left[\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k)\right]. \quad (11)$$

Summing over 0 to $K-1$ and taking the average, we obtain

$$\frac{1}{K}\mathbb{E}\left[L(K)-L(0)\right]+\frac{V}{K}\mathbb{E}\left[\sum_{n\in\mathcal{N}}\omega_n(h_n(K)-h_n(0))\right] < B+V\sum_{n\in\mathcal{N}}\omega_n-\frac{WV}{KN}\mathbb{E}\left[\sum_{k=0}^{K-1}\sum_{n\in\mathcal{N}}\omega_n h_n(k)\right] -\frac{\epsilon}{K}\mathbb{E}\left[\sum_{k=0}^{K-1}\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k)\right]. \quad (12)$$

Let $K\to\infty$, we have

$$\limsup_{K\to\infty}\frac{1}{K}\mathbb{E}\left[\sum_{k=0}^{K-1}\lambda_m(1-\xi_m)Q_m(k)\right] < \frac{B+V\sum_{n\in\mathcal{N}}\omega_n}{\epsilon}, \quad (13)$$

and thus

$$\limsup_{K\to\infty}\frac{1}{K}\mathbb{E}\left[\sum_{k=0}^{K-1}Q_m(k)\right] < +\infty. \quad (14)$$

Therefore, the timely throughput constraint $\{\lambda_m(1-\xi_m)\}$ is satisfied. ■

Remark 1: With similar computation, we have that the weighted average AoI with AAP satisfies

$$\limsup_{K\to\infty}\frac{1}{KN}\mathbb{E}_\pi\left[\sum_{k=0}^{K-1}\sum_{n\in\mathcal{N}}\omega_n h_n(k)\right] < \frac{B}{WV}+\frac{\sum_{n\in\mathcal{N}}\omega_n}{W}. \quad (15)$$

Combining (15) with (13), we find that the upper bound of average virtual length is $O(V)$ while the upper bound of average AoI is $O(1/V)$. This implies a trade-off relation between AoI and the timely-throughput. In the simulation part, we observe that increasing V can decrease the weighted average AoI while the timely throughput constraint is still satisfied. However, the convergence speed of timely throughput gets slower as V increases.

IV. Lower Bound Analysis

In Section III, we develop a scheduling policy that satisfies the timely throughput constraints. However, the upper bound of the weighted average AoI under this policy (15) is not tight. Comparing the average AoI performance of AAP with the minimum weighted average AoI $J(\pi^{\text{Opt}})$ will help in evaluating this policy. But computing $J(\pi^{\text{Opt}})$ directly is difficult due to the curse of dimensionality. So we try to find a lower bound for $J(\pi^{\text{Opt}})$ as a substitute. In this section, we first model the problem of computing a lower bound as a convex optimization problem, and then provide a closed-form lower bound expression when the channel state set satisfies $|\mathcal{C}|=1$. For the general case, an iterative method is introduced to solve this problem.

First, we use $u_n(k)$ to indicate whether user n has updated in the $(k+1)$-th frame or not. $u_n(k)$ is defined as

$$u_n(k) \triangleq \min\left(\sum_{t=0}^{T-1}U_n(kT+t),1\right). \quad (16)$$

The average update frequency of user n in the first K frames is defined as

$$y_n(K) \triangleq \frac{1}{K}\mathbb{E}\left[\sum_{k=0}^{K-1}u_n(k)\right]. \quad (17)$$

For any policy π, it is proved in [9] the weighted average AoI in the first K frames is lower bounded as

$$\frac{1}{K}\mathbb{E}_\pi\left[\sum_{k=0}^{K-1}\sum_{n\in\mathcal{N}}\omega_n h_n(k)\right] \geq \frac{1}{2}\sum_{n\in\mathcal{N}}\frac{\omega_n}{y_{n,\pi}(K)}+\frac{\sum_{n\in\mathcal{N}}\omega_n}{2}. \quad (18)$$

Since the AoI optimization problem (6) is a CMDP, there exists an optimal policy that is stationary randomized policy. Therefore, we can also focus on stationary randomized policy when considering the lower bound (18). If $\pi \in \Pi^{SR}$, the limit $\lim_{K\to\infty}y_n(K)$ exists, which is denoted by $\mu_{a,n}(\pi)$. Then, for $\pi\in\Pi^{SR}$, taking limit $K\to\infty$ in (18) yields,

$$J(\pi) \geq \frac{1}{2N}\sum_{n\in\mathcal{N}}\frac{\omega_n}{\mu_{a,n}(\pi)}+\frac{\sum_{n\in\mathcal{N}}\omega_n}{2N}. \quad (19)$$

With slight abuse of notations, we also use $\boldsymbol{\mu}$ to represent $(\mu_{t,m}, \mu_{a,n} | m \in \mathcal{M}, n \in \mathcal{N})$. The extended achievable region Λ_E is defined similar to that in Definition 1. If $\mu_{a,n}$ is a component of a $\boldsymbol{\mu} \in \Lambda_E$, there exists a policy $\pi \in \Pi^{SR}$ such that

$$\mu_{a,n} \leq \sum_{\boldsymbol{a},\boldsymbol{c}} P(\boldsymbol{a}) P(\boldsymbol{c}) \mathbb{E}_{\pi(\boldsymbol{a},\boldsymbol{c})} \left[\min\left(\sum_{t=0}^{T-1} U_n(t), 1 \right) \middle| \boldsymbol{a}, \boldsymbol{c} \right]. \quad (20)$$

With these preparations, we present the problem of computing AoI lower bound with respect to timely throughput constraints. The optimal value is denoted by J^*_{LB}.

Computing AoI Lower Bound with Timely Throughput Constraint

$$\min_{\pi \in \Pi^{SR}} J_{LB}(\pi) = \frac{1}{2N} \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}(\pi)} + \frac{\sum_{n\in\mathcal{N}} \omega_n}{2N},$$
$$\text{s.t.} \mu_{t,m}(\pi) \geq \lambda_m (1 - \xi_m), \ \forall m \in \mathcal{M}. \quad (21)$$

Because Λ_E also equals the set $\{\boldsymbol{\mu}(\pi) | \pi \in \Pi^{SR}\}$ as shown in Lemma 1, problem (21) is equivalent to the following convex optimization problem,

$$\min_{\boldsymbol{\mu}\in\Lambda_E} P(\boldsymbol{\mu}) = \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}},$$
$$\text{s.t.} \mu_{t,m} \geq \lambda_m(1-\xi_m), \ \forall m \in \mathcal{M}, \quad (22)$$

where $\frac{\sum_{n\in\mathcal{N}} \omega_n}{2N}$ in (21) is omitted since it is fixed.

A. Special Case: $|\mathcal{C}| = 1$

When $|\mathcal{C}| = 1$, the channel condition distribution is time-invariant. In this case, we provide a closed-form expression of the lower bound.

Theorem 2: When $|\mathcal{C}| = 1$, let p_i be the transmission success probability for user i, the minimum weighted average AoI $J(\pi^{\text{Opt}})$ is lower bounded by

$$J(\pi^{\text{Opt}}) \geq \frac{1}{2N\left(T - \sum_{m\in\mathcal{M}} \frac{\lambda_m(1-\xi_m)}{p_m}\right)} \times \left(\sum_{n\in\mathcal{N}} \sqrt{\frac{\omega_n}{p_n}} \right)^2 + \frac{1}{2N} \sum_{n\in\mathcal{N}} \omega_n. \quad (23)$$

Proof: To satisfy the timely throughput constraint, the average number of slots used to service users in $\mathcal{M}$ in each frame is larger than the following term with probability 1,

$$\sum_{m\in\mathcal{M}} \frac{\lambda_m(1-\xi_m)}{p_m}. \quad (24)$$

Therefore, S_{AoI}, the average number of slots in each frame used to service users in $\mathcal{N}$, satisfies,

$$S_{\text{AoI}} \leq T - \sum_{m\in\mathcal{M}} \frac{\lambda_m(1-\xi_m)}{p_m} \quad \text{w.p.1.} \quad (25)$$

On the other hand, we have

$$S_{\text{AoI}} = \sum_{n\in\mathcal{N}} \frac{\mu_{a,n}}{p_n} \quad \text{w.p.1.} \quad (26)$$

Combining these two terms leads to

$$\sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}} \left(T - \sum_{m\in\mathcal{M}} \frac{\lambda_m(1-\xi_m)}{p_m} \right) \geq \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}} \times \sum_{n\in\mathcal{N}} \frac{\mu_{a,n}}{p_n} \geq \left(\sum_{n\in\mathcal{N}} \sqrt{\frac{\omega_n}{p_n}} \right)^2 \quad \text{w.p.1.} \quad (27)$$

The last step is based on *Cauchy-Schwartz* inequality. Dividing both sides by $\left(T - \sum_{m\in\mathcal{M}} \frac{\lambda_m(1-\xi_m)}{p_m}\right)$, we obtain (23) based on (19). ■

B. General Case: $|\mathcal{C}| > 1$

In this part, we study how to compute the weighted average AoI lower bound when the channel condition distribution is time-varying. The starting point is the optimization problem (22), and we use dual decomposition to develop an iterative method. A similar approach is adopted in [20], but it mainly focuses on the case with reliable channels.

We first modify the problem in (22) by introducing an auxiliary variable $\boldsymbol{x} = \{x_n | n \in \mathcal{N}\}$: the problem in (22) is equivalent to

$$\begin{aligned} \min_{\boldsymbol{\mu}\in\Lambda_E, \boldsymbol{x}} \quad & P(\boldsymbol{x}, \boldsymbol{\mu}) = \sum_{n\in\mathcal{N}} \frac{\omega_n}{x_n}, \\ \text{s.t.} \quad & \mu_{t,m} \geq \lambda_m(1-\xi_m), \ \forall m \in \mathcal{M}, \\ & 0 \leq x_n \leq \mu_{a,n}, \ \forall n \in \mathcal{N}. \end{aligned} \quad (28)$$

Let $\boldsymbol{\beta} = (\beta_{t,m}, \beta_{a,n} | m \in \mathcal{M}, n \in \mathcal{N})$ where $\beta_{t,m}$ and $\beta_{a,n}$ are Lagrangian multipliers. The *Lagrangian* of this problem is

$$L(\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\beta}) = \sum_{n\in\mathcal{N}} \frac{\omega_n}{x_n} + \sum_{n\in\mathcal{N}} \beta_{a,n}(x_n - \mu_{a,n}) + \sum_{m\in\mathcal{M}} \beta_{t,m} \left(\lambda_m(1-\xi_m) - \mu_{t,m} \right), \quad (29)$$

and the dual problem is

$$\max_{\boldsymbol{\beta}\geq 0} \min_{\boldsymbol{x}\geq 0, \boldsymbol{\mu}\in\Lambda_E} L(\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\beta}). \quad (30)$$

Because (28) is a convex optimization problem and satisfies *Slater's* condition, strong duality holds and the duality gap is zero [29]. Thus, there exist $\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\beta}^*$ such that,

$$P(\boldsymbol{x}^*, \boldsymbol{\mu}^*) = L(\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\beta}^*). \quad (31)$$

Given $\boldsymbol{\beta}$, the dual problem can be decomposed into the two following subproblems,

$$\text{P3.1} \min_{0\leq \boldsymbol{x}} \sum_{n\in\mathcal{N}} \left(\frac{\omega_n}{x_n} + \beta_{a,n} x_n \right), \quad (32)$$

and

$$\text{P3.2} \max_{\boldsymbol{\mu}\in\Lambda_E} \sum_{n\in\mathcal{N}} \beta_{a,n}\mu_{a,n} + \sum_{m\in\mathcal{M}} \beta_{t,m}\mu_{t,m}. \quad (33)$$

The optimal solution to the P1 is

$$x_n = \sqrt{\frac{\omega_n}{\beta_{a,n}}}, \quad \forall n \in \mathcal{N}, \tag{34}$$

where we allow the denominator $\beta_{a,n}$ to be 0, and in this case $\sqrt{\frac{\omega_n}{\beta_{a,n}}} = +\infty$.

Because this problem is convex, we can use subgradient iteration to obtain the optimal solution. It is obvious that the optimal x_n^* for the primal problem (28) cannot be infinity because $x_n^* \le \mu_{a,n}$ and Λ_E is compact. Thus, we set an upper bound $\overline{X}$ for x_n in the iteration procedure. Let α be the step size. In the k-th iteration, $\boldsymbol{x}$ and $\boldsymbol{\mu}$ are updated as follows,

$$x_n(k) = \min\left(\sqrt{\frac{\omega_n}{\beta_{a,n}(k)}},\ \overline{X}\right),\ \forall n \in \mathcal{N}, \tag{35}$$

$$\boldsymbol{\mu}(k) = \arg\max_{\boldsymbol{\mu}\in\Lambda_E}\left(\sum_{n\in\mathcal{N}}\beta_{a,n}(k)\mu_{a,n} + \sum_{m\in\mathcal{M}}\beta_{t,m}(k)\mu_{t,m}\right), \tag{36}$$

and the *Lagrangian* multipliers are updated as follows,

$$\begin{aligned}\beta_{t,m}(k+1) &= \max(\beta_{t,m}(k) + \alpha\left(\lambda_m(1-\xi_m)\right.\\ &\qquad \left.-\ \mu_{t,m}(k)\right), 0), \end{aligned}\tag{37}$$

$$\begin{aligned}\beta_{a,n}(k+1) &= \max(\beta_{a,n}(k) + \alpha\left(x_n(k)\right.\\ &\qquad \left.-\ \mu_{a,n}(k)\right), 0). \end{aligned}\tag{38}$$

As $k \to \infty$, the iteration converges to the optimal $\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\beta}^*$, and the problem in (22) is solved. However, we hit a snag when trying to compute $\boldsymbol{\mu}(k)$ in (36) because the set Λ_E is complex in the general case. Recalling (4),(5), and (20), we can see that Λ_E is determined by a group of linear inequalities, and the number of variables grows exponentially with M, N, T, which makes it difficult to solve (36). In the next section, we provide an online algorithm to estimate the optimal value of problem (22), whose core idea is to do step (36) by stochastic approximation.

V. Age Oblivious Policy

In this section, we first provide a scheduling policy called *Age-Oblivious Policy* (AOP), inspired by the iteration procedure (35)-(38). This policy tries to minimize the term in (21) instead of the weighted average AoI directly as in (6), and this is where the term *oblivious* comes from. It is interesting to find that AOP can achieve a good weighted average AoI performance, as shown in the simulation. Furthermore, we modify this policy to estimate the optimal value in (22) online and thus avoid solving step (36). The estimation error is also obtained.

AOP is presented in Alg.2. The core variables in this policy update are,

$$\begin{aligned}\beta_{t,m}(k)\ &= \max(\beta_{t,m}(k-1) + \alpha(\lambda_m(1-\xi_m)\\ &\qquad - d_m(k-1)), 0),\end{aligned} \tag{39}$$

$$\begin{aligned}\beta_{a,n}(k)\ &= \max(\beta_{a,n}(k-1) + \alpha(x_n(k-1)\\ &\qquad - d_n(k-1)), 0),\end{aligned} \tag{40}$$

$$x_n(k)\ = \min\left(\sqrt{\frac{\omega_n}{\beta_{a,n}(k)}},\ \overline{X}\right). \tag{41}$$

Algorithm 2 Age-Oblivious Policy (AOP)

Require: Parameter α, $\overline{X}$.
1: $\beta_{a,n}(-1) = 1,\ x_n(-1) = 0,\ h_n(-1) = 0, \forall n \in \mathcal{N}$.
2: $\beta_{t,m}(-1) = 1,\ \forall m \in \mathcal{M};\ k = -1$.
3: **while** 1 **do**
4: $\quad k \leftarrow k + 1$
5: $\quad a_n(k) \leftarrow 1,\ \forall n \in \mathcal{N}$
6: $\quad \beta_{a,n}(k) \leftarrow \beta_{a,n}(k-1) + \alpha x_n(k-1),\ \forall n \in \mathcal{N}$
7: $\quad \beta_{t,m}(k) \leftarrow \beta_{t,m}(k-1) + \alpha\lambda_m(1-\xi_m),\ \forall m \in \mathcal{M}$
8: $\quad$ **for** each slot in the k-th frame **do**
9: $\qquad \mathrm{x}^*, i^* \leftarrow \arg\max_{(\mathrm{x},i)\in\{t,\mathcal{M}\}\cup\{a,\mathcal{N}\}, a_i(k)>0} \beta_{\mathrm{x},i}(k)p_i(c)$
10: $\qquad$ **if** $i^* =$ NULL **then**
11: $\qquad\quad$ Pass
12: $\qquad$ **else**
13: $\qquad\quad$ Schedule user i^*
14: $\qquad\quad$ **if** Transmission succeeds **then**
15: $\qquad\qquad \beta_{\mathrm{x}^*,i^*}(k) \leftarrow \max(\beta_{\mathrm{x}^*,i^*}(k) - \alpha, 0),\ a_{i^*}(k) \leftarrow a_{i^*}(k) - 1$
16: $\qquad\qquad$ **if** $i^* \in \mathcal{N}$ **then**
17: $\qquad\qquad\quad h_{i^*}(k) \leftarrow 0$
18: $\qquad\qquad$ **end if**
19: $\qquad\quad$ **end if**
20: $\qquad$ **end if**
21: $\quad$ **end for**
22: $\quad x_n(k) \leftarrow \min\left(\sqrt{\frac{\omega_n}{\beta_{a,n}(k)}},\ \overline{X}\right),\ \forall n \in \mathcal{N}$
23: **end while**

Recalling that $d_m(k)$ is the number of delivered timely throughput packets for user m in the $(k+1)$-th frame, and $u_n(k)$ is the update indicator for user n in the $(k+1)$-th frame, we can interpret $\beta_{t,m}(k)$ and $\beta_{a,n}(k)$ as virtual queues. The evolution of $\beta_{t,m}(k)$ is the same as (7) except for the parameter α. As for virtual queue $\beta_{a,n}(k)$, its arrival is $x_n(k)$ and departure is $u_n(k)$. One difference is that $x_n(k)$ is adjustable as shown in (41).

We first prove that this policy satisfies the timely throughput constraint.

Theorem 3: AOP satisfies all timely throughput constraints $\{\lambda_m(1-\xi_m)\}$ in the interior of Λ.

Proof: Because $\beta_{t,m}(k)$ can be interpreted as a virtual queue for timely throughput, we can prove this theorem by showing that

$$\limsup_{K\to\infty}\frac{1}{K}\sum_{k=0}^{K-1}\sum_{m\in\mathcal{M}}\mathbb{E}[\beta_{t,m}(k)] < +\infty. \tag{42}$$

See detail in Appendix B. ■

Line 9 in Alg.2 is in fact a kind of *stochastic approximation* [30], during which we solve (36) on every sample path of transmission result $\{\boldsymbol{U}(0), \cdots, \boldsymbol{U}(T-1)\}$. Since AOP resembles iteration steps (35)-(38), it is natural to ask: can we modify Alg.2 to solve problem (21)? The following theorem shows that we can get a good estimation of the optimal value J_{LB}^* in problem (21), i.e., the lower bound of minimum AoI.

Theorem 4: If the timely throughput constraints $\{\lambda_m(1-\xi_m)\}$ are in the interior of Λ, we have

$$\limsup_{K\to\infty}\sum_{n\in\mathcal{N}}\frac{\omega_n K}{2N\mathbb{E}\left[\sum_{k=0}^{K-1}x_n(k)\right]}+\frac{\sum_{n\in\mathcal{N}}\omega_n}{2N} \leq \frac{\alpha B}{2N}+J^*_{\text{LB}}, \quad (43)$$

where

$$B \triangleq \frac{1}{2}\left(\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+N\overline{X}^2+T^2\right). \quad (44)$$

Proof: See Appendix B. ■

Therefore, the following expression serves as a good estimation of the optimal value in (21),

$$\limsup_{K\to\infty}\sum_{n\in\mathcal{N}}\frac{\omega_n K}{2N\mathbb{E}\left[\sum_{k=0}^{K-1}x_n(k)\right]}+\frac{\sum_{n\in\mathcal{N}}\omega_n}{2N}. \quad (45)$$

And the gap is at most

$$\frac{\alpha B}{2N}=\frac{\alpha}{4N}\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+\frac{\alpha}{4N}T^2+\frac{\alpha}{4}\overline{X}^2. \quad (46)$$

VI. Simulation Results

In the simulation part, we are interested in the following issues: 1) the accuracy of the online lower bound estimation method; 2) the performance of AAP and AOP; 3) the influence of parameter V; 4) the influence of parameter α. If not mentioned specifically, the simulation horizon is 3×10^6 frames, and the length of a frame is 5 time-slots. Each data point is obtained by averaging 10 simulation results. For simplicity, we call the users in set $\mathcal{N}$ *age user* and the users in set $\mathcal{M}$ *timely user*. We use the sample mean of (45), called Lower Bound$_{\text{AOP}}$, to estimate the lower bound J^*_{LB}. $\overline{X}$ equals 1 in the simulation.

A. Special Case: $|\mathcal{C}|=1$

In this case, the channel condition distribution is fixed, and thus we can obtain the closed-form expression of lower bound based on (23). The number of age users is 20. Half of the users are assigned with weight 0.8 and the channel reliability is 0.8. For the other half, their weight is 1 and the channel reliability is 0.5. The number of timely users is 10. The arrival process follows Bernoulli distribution. Half of the users have arrival rate 0.4 and their channel reliability is 0.8. For the rest, the arrival rate is 0.3 and channel reliability is 0.6. All timely users have the same maximum allowable drop rate ξ.

In Fig. 3, we investigate the accuracy of the online estimation method. Drop rate ξ is 0.25. As α becomes larger, we can see that Lower Bound$_{\text{AOP}}$ is very close to the Lower Bound given by (23). When $\alpha<0.1$, the estimation Lower Bound$_{\text{AOP}}$ is smaller than the theoretical lower bound, this is mainly because AOP has not converged yet, as we will see in Fig. 9. This result further justifies using Lower Bound$_{\text{AOP}}$ as a substitute for the theoretical lower bound in (21).

In Fig. 4, we compare the weighted average AoI of AAP and AOP. In this simulation, $\alpha=1$ and $V=1$. Drop rate ξ

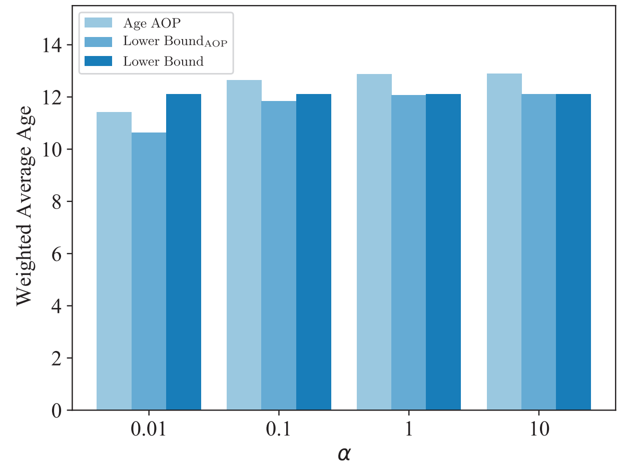

Fig. 3. The influence of α on the accuracy of online lower bound estimation.

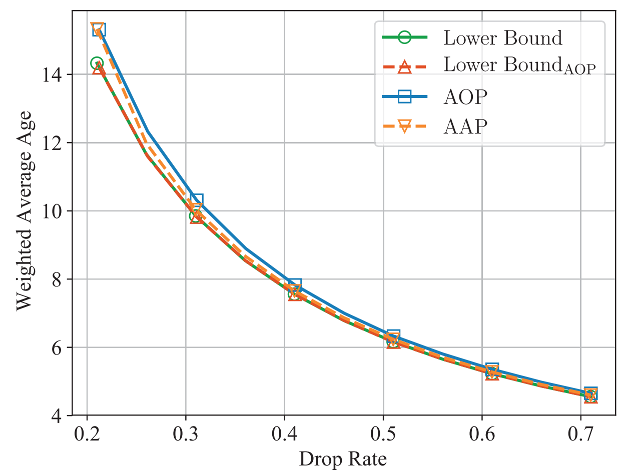

Fig. 4. AAP and AOP's performance with respect to varying drop rate in the special case.

starts at 0.21, which is very close to the boundary of the achievable region. The gap between the weighted average AoI of AAP and that of AOP are very narrow even when the ξ is small, i.e., when the timely throughput constraint is strict. Their distance to the lower bound shrinks when the constraint becomes less strict. Besides, the curve of Lower Bound$_{\text{AOP}}$ overlaps the curve of the theoretical lower bound in the whole drop rate region, which corroborates our findings in Fig. 3.

B. General Case: $|\mathcal{C}|>1$

For this case, we assume that the channel has two states, and each happens with probability 0.5. There are two types of age users, each consists of 10 users. Type-1 age users' weight is 0.8 and type-2 age users' weight is 1. Timely users are also divided into two types, each has 5 users. The arrival process follows Bernoulli distribution. The arrival rate is 0.4 for type-1, and 0.3 for type-2. When the channel is in state **1**, the channel reliability is 0.9 for the type-1 user, and 0.4 for the type-2 user, either age user or timely user. When the channel is in state **2**,

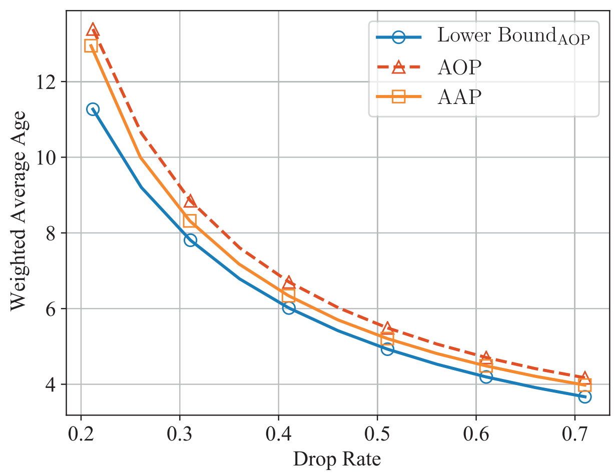

Fig. 5. AAP and AOP's performance with respect to varying drop rate in general case. The estimation error $\alpha B/2N$ is from 0.56 to 0.57.

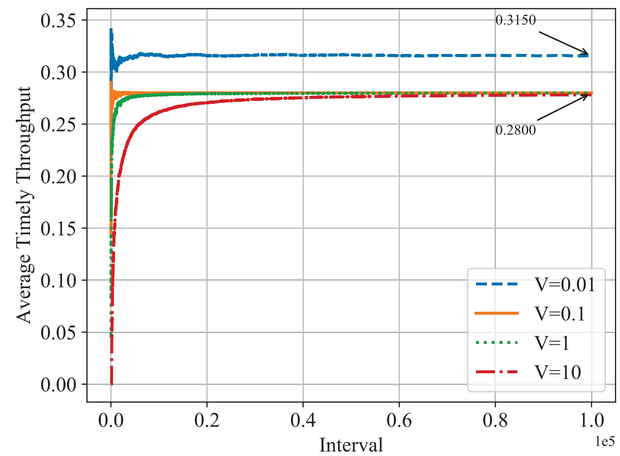

Fig. 7. Influence of V on the timely throughput convergence rate of AAP.

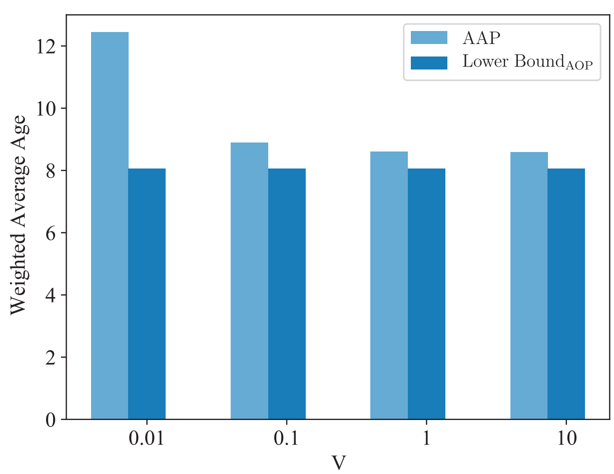

Fig. 6. Influence of V on the average AoI of AAP.

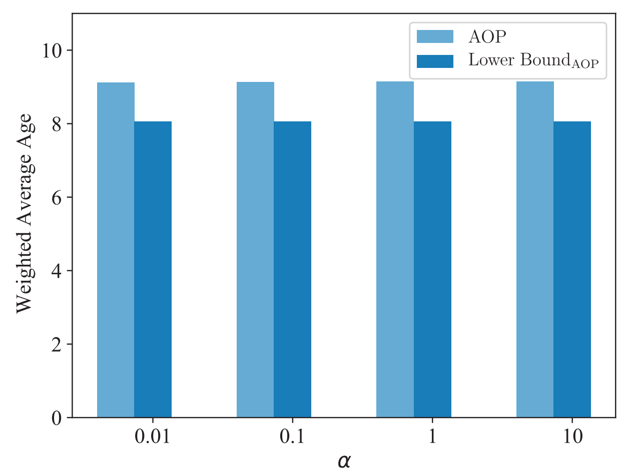

Fig. 8. Influence of α on the average AoI of AOP.

the channel reliability is 0.5 for type-1 age user and is 0.7 for type-2 age user. For the timely user, both types have channel reliability 0.7.

In Fig. 5, we compare the weighted average AoI of AAP and AOP like that in Fig. 4. In this simulation, $\alpha = 1$ and $V = 1$. We evaluate performance by comparing it to Lower Bound$_{\text{AOP}}$. In this case, the estimation error $\alpha B/N$ is just about 0.56. Both policies' distance to Lower Bound$_{\text{AOP}}$ decreases as the drop rate ξ increases and the gap is relatively small.

A problem with AAP is that this policy may service timely throughput traffic excessively when V is small, and thus degrade the weighted average AoI performance, as shown in Fig. 6 and Fig. 7. The drop rate requirement ξ is 0.3. In Fig. 6, we study the influence of parameter V on the resulting average AoI. In Fig. 7, we take type-1 timely user as a sample to study the influence of V on the convergence rate of timely throughput. The Y-axis is the finite horizon average of timely throughput. As we can see in Fig. 6, increasing V leads to smaller average AoI, which is consistent with the upper bound in (15). Furthermore, when $V = 0.01$, the average AoI is larger than Lower Bound$_{\text{AOP}}$ by more than 40%. And in Fig. 7, when $V = 0.01$, the $\mu_{t,m} = 0.315$ packets per frame, larger than requirement 0.28 packets per frame by 12.5%. This phenomenon may stem from the residual in the virtual queue. In (7), the queue length reduction is integer while the increment is a real number, and thus the virtual queue length might be less than one (residual). Since AAP makes decisions by comparing the virtual queue length and AoI multiplied by V, the timely user may receive excessive service when V is small.

In Fig. 8 and Fig. 9, we study the influence of α on the average AoI and the convergence rate of timely throughput of AOP. In this case, we find changing α has almost no influence on the average AoI. As for the timely throughput, we also take type-1 timely user as a sample. When α increases, the convergence rate speeds up, but AOP does not exhibit excessive service behavior like AAP. In contrast to that in AAP, these findings suggest that we can use large step size α without worrying about impairing the average AoI.

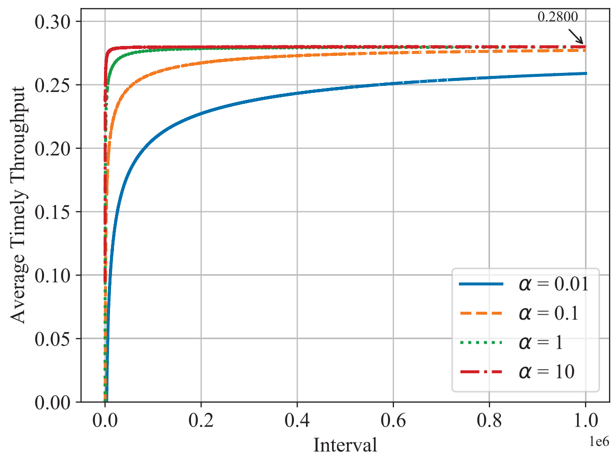

Fig. 9. Influence of α on the timely throughput convergence rate of AOP.

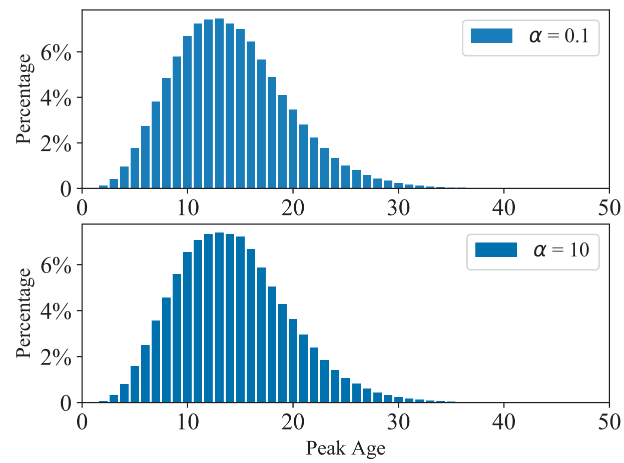

Fig. 11. Distribution of peak AoI under AOP.

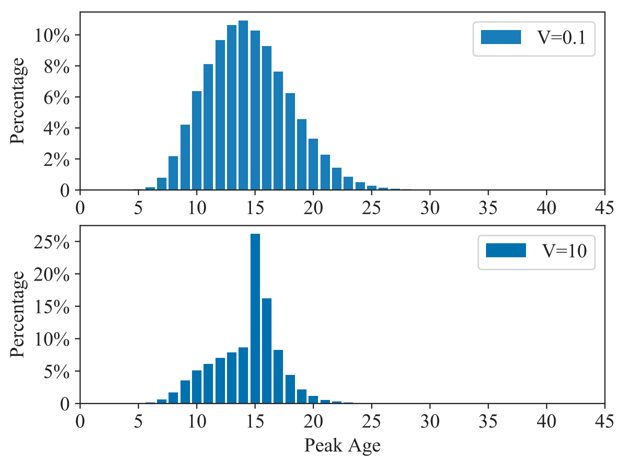

Fig. 10. Distribution of peak AoI under AAP.

For status update traffic, besides the average AoI, we are also interested in the *peak AoI*, which is defined as the AoI just before an update happens. We investigate the influence of V and α on the distribution of peak AoI of Type-1 age user. The drop rate requirement is 0.35 in this simulation. In Fig. 10, we find that the variance of peak AoI decreases when V increases. This may be explained that increasing V is equivalent to increasing the policy decision's sensitivity to AoI: when V is large, once a user's AoI is larger than a threshold, the decision in a foreseeable future is to schedule this user until the AoI drops. The larger V is, the quicker to schedule. In contrast, we find that tuning α has little influence on the peak AoI distribution as shown in Fig. 11.

In summary, simulation results show that: 1) the online estimation method yields result very close to the theoretical lower bound; 2) when V is small, the timely throughput under AAP converges faster at the price of a worse weighted average AoI; 3) the advantage of AOP over AAP is that AOP does not show excessive service behavior like AAP; 4) AAP can have better peak AoI performance than AOP.

VII. Conclusion

In this work, we studied the problem of minimizing weighted average AoI subject to timely throughput constraints. Two scheduling policies were proposed: the age-aware policy makes decision based on current AoI, while the age-oblivious policy does not. While both policies satisfy all timely throughput constraints in the achievable region, their gap to the minimum weighted average AoI is unknown. Because of this, we developed an online method to estimate a lower bound of the minimum average AoI based on dual decomposition. Simulation results show that the average AoI performance of the age-oblivious policy is comparable to that of the age-aware policy, and both are close to the lower bound. However, in terms of peak AoI, the age-aware policy's performance is adjustable by tuning the policy parameter, while age-oblivious policy cannot. It must be noted that in this work, we assumed that there is a BS having complete knowledge of all channel states and packet arrivals information. The assumption of such a central controller may be impractical in real system when the number of users is large. In future work, we will consider the distributed access protocol when there is no central controller. Since the virtual queues of age-oblivious policy change smoothly,[4] it is possible to design distributed access based on the queue length like Q-CSMA [31]. Also, a more general timely throughput model should be explored.

Appendix A
Proof for Theorem 1

Let $\mathcal{F}(k) \triangleq \{\boldsymbol{Q}(k), \boldsymbol{H}(k), \boldsymbol{a}(k), \boldsymbol{c}(k)\}$. Taking expectation under AAP, we consider drift $\Delta(k)$, which is defined as

$$\Delta(k) \triangleq \mathbb{E}[L(k+1) - L(k)|\mathcal{F}(k)]. \tag{47}$$

Since $\max(x, 0) \leq x^2$, we have

$$Q_m^2(k+1) - Q_m^2(k) \leq 2\, Q_m(k) \left(\lambda_m(1-\xi_m) - d_m(k)\right) + \left(\lambda_m(1-\xi_m) - d_m(k)\right)^2,$$

[4]one difficulty with design AoI-optimal distributed access protocol is that the change of AoI is not smooth, e.g., jump to 0 once updated if the source is active.

and thus the drift can be expanded into the following expression,

$$\begin{aligned}\Delta(k) \leq\ & \frac{1}{2}\mathbb{E}\left[\sum_{m\in\mathcal{M}}\left(\lambda_m(1-\xi_m)-d_m(k)\right)^2|\mathcal{F}(k)\right] \\ &+\sum_{m\in\mathcal{M}}Q_m(k)\left(\lambda_m(1-\xi_m)-\mathbb{E}[d_m(k)|\mathcal{F}(k)]\right) \\ \leq\ & \frac{1}{2}\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+\frac{1}{2}\mathbb{E}\left[\sum_{m\in\mathcal{M}}d_m(k)^2\right] \\ &+\sum_{m\in\mathcal{M}}Q_m(k)\left(\lambda_m(1-\xi_m)-\mathbb{E}[d_m(k)|\mathcal{F}(k)]\right) \\ \leq\ & \frac{1}{2}\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+\frac{1}{2}\mathbb{E}\left[\left(\sum_{m\in\mathcal{M}}d_m(k)\right)^2\right] \\ &+\sum_{m\in\mathcal{M}}Q_m(k)\left(\lambda_m(1-\xi_m)-\mathbb{E}[d_m(k)|\mathcal{F}(k)]\right) \\ \leq\ & \frac{1}{2}\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+\frac{1}{2}T^2 \\ &+\sum_{m\in\mathcal{M}}Q_m(k)\left(\lambda_m(1-\xi_m)-\mathbb{E}[d_m(k)|\mathcal{F}(k)]\right).\end{aligned} \tag{48}$$

Let

$$B \triangleq \frac{1}{2}\sum_{m\in\mathcal{M}}\lambda_m^2(1-\xi_m)^2+T^2. \tag{49}$$

Replacing $d_m(k)$ by its definition in (2) yields,

$$\begin{aligned}\Delta(k) \leq\ & B+\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k)-\sum_{m\in\mathcal{M}}Q_m(k)\mathbb{E} \\ &\times\left[\min\left(\sum_{t=0}^{T-1}U_m(kT+t),a_m\right)\middle|\mathcal{F}(k)\right].\end{aligned} \tag{50}$$

The conditional expectation of weighted Age is defined as

$$\begin{aligned}P(k) \triangleq\ & \mathbb{E}\left[\sum_{n\in\mathcal{N}}\omega_n h_n(k+1)\middle|\mathcal{F}(k)\right] \\ =\ & \sum_{n\in\mathcal{N}}\omega_n h_n(k)+\sum_{n\in\mathcal{N}}\omega_n-\sum_{n\in\mathcal{N}}\omega_n h_n(k)\mathbb{E} \\ &\times\left[\mathbb{I}\left(\sum_{t=0}^{T-1}U_n(kT+t)>0\right)\middle|\mathcal{F}(k)\right].\end{aligned} \tag{51}$$

Adding $\Delta(k)$ and $VP(k)$, we obtain

$$\begin{aligned}\Delta(k)+VP(k) \leq\ & B+\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k) \\ &+V\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1)-\sum_{m\in\mathcal{M}}Q_m(k)\mathbb{E} \\ &\times\left[\min\left(\sum_{t=0}^{T-1}U_m(kT+t),a_m\right)\middle|\mathcal{F}(k)\right] \\ &-V\sum_{n\in\mathcal{N}}\omega_n h_n(k)\mathbb{E} \\ &\times\left[\mathbb{I}\left(\sum_{t=0}^{T-1}U_n(kT+t)>0\right)\middle|\mathcal{F}(k)\right].\end{aligned} \tag{52}$$

Because $\{\lambda_m(1-\xi_m)(1+2\epsilon)\}\in\Lambda$, there exists a policy $\pi_1\in\Pi^{SR}$ whose timely throughput $\boldsymbol{\mu}(\pi_1)$ is exactly $\{\lambda_m(1-\xi_m)(1+2\epsilon)\}$. To proceed, we define a pure AoI policy π_2 that randomly schedules a user in $\mathcal{N}$ with equal probability in each time slot. Then, let π be a probabilistic mixture of π_1 and π_2,

$$\pi=\frac{1+\epsilon}{1+2\epsilon}\pi_1+\frac{\epsilon}{1+2\epsilon}\pi_2, \tag{53}$$

which means, at the beginning of each frame, we choose to use policy π_1 with probability $\frac{1+\epsilon}{1+2\epsilon}$ and policy π_2 with probability $\frac{\epsilon}{1+2\epsilon}$. Since Λ is convex, we obtain $\boldsymbol{\mu}(\pi)=\{\lambda_m(1-\xi_m)(1+\epsilon)\}$. Furthermore, each user in $\mathcal{N}$ is scheduled with probability $\frac{\epsilon}{(1+2\epsilon)N}$ in each time slot under policy π. Therefore,

$$\mathbb{E}_\pi\left[\mathbb{I}\left(\sum_{t=0}^{T-1}U_n(kT+t)>0\right)\middle|\mathcal{F}(k)\right]>p_n(\boldsymbol{c})\frac{\epsilon}{(1+2\epsilon)N}. \tag{54}$$

Note that the Alg.1 decides to minimize the sum of the last two terms on the RHS of (52) on every sample path of the transmission result $\{\boldsymbol{U}(0),\cdots,\boldsymbol{U}(T-1)\}$, and thus Alg.1 minimizes the expectation as well. Hence, with Alg.1, we have

$$\begin{aligned}&\Delta(k)+VP(k) \\ &\leq B+\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k)+V\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1) \\ &\quad-\sum_{m\in\mathcal{M}}Q_m(k)\mathbb{E}_\pi\left[\min\left(\sum_{t=0}^{T-1}U_m(kT+t),a_m\right)\middle|\mathcal{F}(k)\right] \\ &\quad-V\sum_{n\in\mathcal{N}}\omega_n h_n(k)\mathbb{E}_\pi\left[\mathbb{I}\left(\sum_{t=0}^{T-1}U_n(kT+t)>0\right)\middle|\mathcal{F}(k)\right] \\ &< B+\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k)+V\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1) \\ &\quad-V\sum_{n\in\mathcal{N}}\omega_n h_n(k)p_n(\boldsymbol{c})\frac{\epsilon}{(1+2\epsilon)N} \\ &\quad-\sum_{m\in\mathcal{M}}Q_m(k)\mu_{t,m}(\pi,\boldsymbol{a},\boldsymbol{c}),\end{aligned} \tag{55}$$

where

$$\mu_{t,m}(\pi,\boldsymbol{a},\boldsymbol{c})\triangleq\mathbb{E}_\pi\left[\min\left(\sum_{t=0}^{T-1}U_m(kT+t),a_m\right)\middle|\mathcal{F}(k)\right]. \tag{56}$$

Taking expectation w.r.t. $\boldsymbol{a}$ and $\boldsymbol{c}$ on both sides of (55) leads to

$$\begin{aligned}&\mathbb{E}\left[L(k+1)-L(k)+V\sum_{n\in\mathcal{N}}\omega_n h_n(k+1)\middle|\boldsymbol{Q}(k),\boldsymbol{H}(k)\right] \\ &< B+\sum_{m\in\mathcal{M}}\lambda_m(1-\xi_m)Q_m(k)+V\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1) \\ &\quad-\frac{WV}{N}\sum_{n\in\mathcal{N}}\omega_n h_n(k)-\sum_{m\in\mathcal{M}}Q_m(k)\lambda_m(1-\xi_m)(1+\epsilon) \\ &= B+V\sum_{n\in\mathcal{N}}\omega_n(h_n(k)+1)-\frac{WV}{N}\sum_{n\in\mathcal{N}}\omega_n h_n(k) \\ &\quad-\epsilon\sum_{m\in\mathcal{M}}Q_m(k)\lambda_m(1-\xi_m),\end{aligned} \tag{57}$$

where $W\triangleq\min_{n\in\mathcal{N}}\mathbb{E}_{\boldsymbol{c}}[p_n(\boldsymbol{c})\frac{\epsilon}{(1+2\epsilon)}]$. ■

APPENDIX B
PROOF FOR THEOREM 3 AND 4

We first state the following lemma.

Lemma 2: Let $\boldsymbol{\mu}^*$ be the optimal solution to problem (22). For any $\epsilon > 0$, there exists $\boldsymbol{\mu}^\epsilon \in \Lambda_E$ such that

$$P^* \triangleq \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu^*_{a,n}} \geq \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu^\epsilon_{a,n}} - \delta(\epsilon), \tag{58}$$

$$\mu^\epsilon_{t,m} \geq \lambda_m(1-\xi_m) + \epsilon,\ \forall m \in \mathcal{M}, \tag{59}$$

where $\delta(\epsilon)$ is positive and $\lim_{\epsilon\to 0}\delta(\epsilon) = 0$.

Proof: Let

$$\Lambda_1 \triangleq \{\boldsymbol{\mu} \in \Lambda_E | \mu_{t,m} \geq \lambda_m(1-\xi_m),\ \forall m \in \mathcal{M}\}, \tag{60}$$

$$\Lambda_2 \triangleq \{\boldsymbol{\mu} \in \Lambda_E | \mu_{t,m} \geq \lambda_m(1-\xi_m) + \epsilon,\ \forall m \in \mathcal{M}\}. \tag{61}$$

Then we have

$$\boldsymbol{\mu}^* = \underset{\boldsymbol{\mu}\in\Lambda_E\cap\Lambda_1}{\arg\min} \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}}. \tag{62}$$

Let

$$\boldsymbol{\mu}^\epsilon = \underset{\boldsymbol{\mu}\in\Lambda_E\cap\Lambda_2}{\arg\min} \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}}, \tag{63}$$

then condition (59) is satisfied. Because $\Lambda_2 \subset \Lambda_1$, condition (58) is also met. Finally, since the optimization target is a continuous function on Λ_E, we conclude that $\lim_{\epsilon\to 0}\delta(\epsilon) = 0$. ■

Based on Lemma 2, there exists a policy $\pi \in \Pi^{SR}$ such that

$$\sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu_{a,n}(\pi)} \leq \sum_{n\in\mathcal{N}} \frac{\omega_n}{\mu^*_{a,n}} + \delta(\epsilon), \tag{64}$$

$$\mu_{t,m}(\pi) \geq \lambda_m(1-\xi_m) + \epsilon,\ \forall m \in \mathcal{M}. \tag{65}$$

To proceed, we define the quadratic Lyapunov function

$$L(k) \triangleq \frac{1}{2}\sum_{n\in\mathcal{N}} \beta^2_{a,n}(k) + \frac{1}{2}\sum_{m\in\mathcal{M}} \beta^2_{t,m}(k), \tag{66}$$

and the drift[5] is

$$\begin{aligned}\Delta(k) &= \mathbb{E}[L(k+1) - L(k)|\boldsymbol{\beta}(k)] \\ &\leq \frac{\alpha^2}{2}\mathbb{E}\left[\sum_{m\in\mathcal{M}} (\lambda_m(1-\xi_m) - d_m(k))^2 \middle| \boldsymbol{\beta}(k)\right] \\ &\quad + \frac{\alpha^2}{2}\mathbb{E}\left[\sum_{n\in\mathcal{N}} (x_n(k) - u_n(k))^2 \middle| \boldsymbol{\beta}(k)\right] \\ &\quad + \alpha\sum_{m\in\mathcal{M}} \lambda_m(1-\xi_m)\beta_{t,m}(k) + \alpha\sum_{n\in\mathcal{N}} x_n(k)\beta_{a,n}(k) \\ &\quad - \alpha\mathbb{E}\left[\sum_{n\in\mathcal{N}} \beta_{a,n}(k)u_n(k)\right. \\ &\quad \left. + \sum_{m\in\mathcal{M}} \beta_{t,m}(k)d_m(k) \middle| \boldsymbol{\beta}(k)\right].\end{aligned} \tag{67}$$

[5]In this section, if not mention specifically, expectation is taken under scheduling policy AOP.

Similar to (48), we have

$$\begin{aligned}\Delta(k) &\leq \alpha^2 B + \alpha\sum_{m\in\mathcal{M}} \lambda_m(1-\xi_m)\beta_{t,m}(k) \\ &\quad + \alpha\sum_{n\in\mathcal{N}} x_n(k)\beta_{a,n}(k) \\ &\quad - \alpha\mathbb{E}\left[\sum_{n\in\mathcal{N}} \beta_{a,n}(k)u_n(k)\right. \\ &\quad \left. + \sum_{m\in\mathcal{M}} \beta_{t,m}(k)d_m(k) \middle| \boldsymbol{\beta}(k)\right],\end{aligned} \tag{68}$$

where,

$$B \triangleq \frac{1}{2}\left(\sum_{m\in\mathcal{M}} \lambda^2_m(1-\xi_m)^2 + N\overline{X}^2 + T^2\right). \tag{69}$$

Since Alg.2 minimizes the last term in the RHS of (68), using policy π in the k-th frame yields

$$\begin{aligned}\Delta(k) &\leq \alpha^2 B + \alpha\sum_{m\in\mathcal{M}} \lambda_m(1-\xi_m)\beta_{t,m}(k) \\ &\quad + \alpha\sum_{n\in\mathcal{N}} x_n(k)\beta_{a,n}(k) \\ &\quad - \alpha\sum_{n\in\mathcal{N}} \mu_{a,n}(\pi)\beta_{a,n}(k) - \alpha\sum_{m\in\mathcal{M}} \mu_{t,m}(\pi)\beta_{t,m}(k) \\ &= \alpha^2 B - \epsilon\alpha\sum_{m\in\mathcal{M}} \beta_{t,m}(k) \\ &\quad + \alpha\sum_{n\in\mathcal{N}} \left(x_n(k)\beta_{a,n}(k) + \frac{\omega_n}{x_n(k)}\right) \\ &\quad - \alpha\sum_{n\in\mathcal{N}} \left(\mu_{a,n}(\pi)\beta_{a,n}(k) + \frac{\omega_n}{\mu_{a,n}(\pi)}\right) \\ &\quad + \alpha\sum_{n\in\mathcal{N}} \left(\frac{\omega_n}{\mu_{a,n}(\pi)} - \frac{\omega_n}{x_n(k)}\right).\end{aligned} \tag{70}$$

Because

$$x_n(k) = \min\left(\sqrt{\frac{\omega_n}{\beta_{a,n}(k)}},\ \overline{X}\right),\ \forall n \in \mathcal{N}, \tag{71}$$

we have

$$\alpha\sum_{n\in\mathcal{N}} \left(x_n(k)\beta_{a,n}(k) + \frac{\omega_n}{x_n(k)}\right) \leq \alpha\sum_{n\in\mathcal{N}} \left(\mu_{a,n}\beta_{a,n}(k) + \frac{\omega_n}{\mu_{a,n}}\right). \tag{72}$$

Therefore

$$\begin{aligned}\Delta(k) \leq\ & \alpha^2 B - \epsilon\alpha\sum_{m\in\mathcal{M}} \beta_{t,m}(k) \\ & + \alpha\sum_{n\in\mathcal{N}} \left(\frac{\omega_n}{\mu_{a,n}(\pi)} - \frac{\omega_n}{x_n(k)}\right)\end{aligned} \tag{73}$$

Taking expectation on both sides, summing over 0 to $K-1$, taking the average, and letting $K \to \infty$, similar to the proof of Corollary 1, we obtain

$$\begin{aligned}\limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1}\sum_{m\in\mathcal{M}} \mathbb{E}\left[\beta_{t,m}(k)\right] &\leq \frac{\alpha B}{\epsilon} + \frac{P^* + \delta(\epsilon)}{\epsilon} \\ &< +\infty,\end{aligned} \tag{74}$$

which proves Theorem 3.

Summing over 0 to $K-1$ in (73) taking the average, we obtain

$$\begin{aligned}\frac{1}{K}\mathbb{E}\left[L(K)-L(0)\right] \leq\ & \alpha^2 B+\alpha(P^*+\delta(\epsilon)) \\ & -\frac{\alpha}{K}\sum_{k=0}^{K-1}\sum_{n\in\mathcal{N}}\mathbb{E}\left[\frac{\omega_n}{x_n(k)}\right] \\ \overset{(a)}{\leq}\ & \alpha^2 B+\alpha(P^*+\delta(\epsilon)) \\ & -\alpha\sum_{n\in\mathcal{N}}\frac{\omega_n K}{\mathbb{E}\left[\sum_{k=0}^{K-1}x_n(k)\right]}, \quad (75)\end{aligned}$$

where (a) is based on Jensen's Inequality and that $\frac{1}{x}$ is convex when $x>0$.

Let $K\to\infty$, we have

$$\limsup_{K\to\infty}\sum_{n\in\mathcal{N}}\frac{\omega_n K}{\mathbb{E}\left[\sum_{k=0}^{K-1}x_n(k)\right]}\leq \alpha B+P^*. \quad (76)$$

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Jingzhou Sun received the B.E. degree from the Department of Electronic Engineering, Tsinghua University, in 2018, where he is currently pursuing the Ph.D. degree with the Network Integration for Ubiquitous Linkage and Broadband Laboratory (NIULAB). His research interests include intelligent resource allocation in wireless networks, low-latency wireless communications, and massive access technology.

Lehan Wang received the B.E. degree from the School of Information and Communication Engineering, University of Electronic Science and Technology of China, in 2019. She is currently pursuing the Ph.D. degree with the Network Integration for Ubiquitous Linkage and Broadband Laboratory (NIULAB), Department of Electronic Engineering, Tsinghua University. Her research interests include performance evaluation of low-latency communication systems and online learning.

Zhiyuan Jiang (Member, IEEE) received the B.S. and Ph.D. degrees from the Department of Electronic Engineering, Tsinghua University, China, in 2010 and 2015, respectively. He visited the WiDeS Group, University of Southern California, Los Angeles, CA, USA, from 2013 to 2014. He worked as an Experienced Researcher with Ericsson from 2015 to 2016. He visited ARNG, University of Southern California from 2017 to 2018. He worked as a Wireless Signal Processing Scientist with Intel Labs, Hillsboro, OR, USA, from 2018. He is currently working as a Professor with the School of Communication and Information Engineering, Shanghai University, Shanghai, China. His current research interests include URLLC in wireless networked control systems and signal processing in MIMO systems. He serves as a TPC Member for IEEE INFOCOM, ICC, GLOBECOM, and WCNC. He received the ITC Rising Scholar Award in 2020, the Best Paper Award from the IEEE ICC 2020, the Best in-Session Presentation Award from the IEEE INFOCOM 2019, and an Exemplary Reviewer Award from IEEE WCL in 2019. He serves as an Associated Editor for IEEE/KICS JOURNAL OF COMMUNICATIONS AND NETWORKS and a Guest Editor for IEEE INTERNET OF THINGS JOURNAL.

Sheng Zhou (Member, IEEE) received the B.E. and Ph.D. degrees in electronic engineering from Tsinghua University, Beijing, China, in 2005 and 2011, respectively. In 2010, he was a Visiting Student with the Wireless System Laboratory, Department of Electrical Engineering, Stanford University, Stanford, CA, USA. From 2014 to 2015, he was a Visiting Researcher with the Central Research Laboratory, Hitachi Ltd., Japan. He is currently an Associate Professor with the Department of Electronic Engineering, Tsinghua University. His research interests include cross-layer design for multiple antenna systems, mobile edge computing, vehicular networks, and green wireless communications. He received the IEEE ComSoc Asia–Pacific Board Outstanding Young Researcher Award in 2017.

Zhisheng Niu (Fellow, IEEE) received the B.E. degree from Beijing Jiaotong University, China, in 1985, and the M.E. and D.E. degrees from the Toyohashi University of Technology, Japan, in 1989 and 1992, respectively.

From 1992 to 1994, he was with Fujitsu Laboratories Ltd., Japan. In 1994, he joined Tsinghua University, Beijing, China, where he is currently a Professor with the Department of Electronic Engineering. His major research interests include queuing theory, traffic engineering, mobile Internet, radio resource management of wireless networks, and green communication and networks. He is a fellow of IEICE. He received the Outstanding Young Researcher Award from the Natural Science Foundation of China in 2009 and the Best Paper Award from the IEEE Communication Society Asia–Pacific Board in 2013. He has served as the Chair for the Emerging Technologies Committee from 2014 to 2015, the Director for the Conference Publications from 2010 to 2011, and the Director for the Asia–Pacific Board in the IEEE Communication Society from 2008 to 2009. He is currently serving as the Director for Online Contents from 2018 to 2019 and an Area Editor for the IEEE TRANSACTIONS ON GREEN COMMUNICATIONS AND NETWORKING. He was also selected as a Distinguished Lecturer of the IEEE Communications Society from 2012 to 2015 and the IEEE Vehicular Technology Society from 2014 to 2018.