Optimal Recursive Power Allocation for Energy Harvesting System With Multiple Antennas

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***Abstract*—In this paper, we investigate the optimal recursive power-allocation policies with energy harvesting wireless nodes equipped with multiple antennas in a fading channel. This optimization problem includes several complex matrices as optimization variables. As a difference, existing optimization theory and methods have been designed to solve these problems over real space. Naturally, the optimization variables have been assumed points in the real space. We proposed a transform approach and designed the algorithms for solving the throughput maximization problem and transmission completion time minimization problem for a multiple-input–multiple-output (MIMO) system. The algorithms were further extended to solve the throughput maximization problem of a hybrid system with both harvesting energy and grid power. Numerical results illustrated the algorithm steps and significant efficiency of the proposed algorithms. To the best of the authors' knowledge, there are no existing algorithms reported in the open literature to obtain exact solutions to the proposed problems. Significant features of the proposed algorithms are that 1) they provide the exact optimal solutions via efficient finite computation and that 2) optimality of the proposed algorithms is strictly proven.**

***Index Terms*—Energy harvesting, exact solution, geometric water-filling (GWF), low-degree polynomial complexity, multiple-input–multiple-output (MIMO), optimization in complex matrices, power grid, radio resource management (RRM).**

I. Introduction

PROLONGING the lifetime of the batteries in wireless communication systems is extremely important. One possible technique to overcome the limitation of battery lifetime is to harvest energy from the environment, such as vibration absorption devices, solar energy, wind energy, thermal energy, and other clean energy [1]. In such systems, energy harvesting has become a preferred choice for supporting "green communication." Furthermore, the multiple-input–multiple-output (MIMO) technology [2] uses multiple antennas at either the transmitter or the receiver or both sides to significantly increase data throughput and link range without additional bandwidth or transmitted power. Thus, it plays an important role in advanced wireless communication systems. The focus of this paper is on investigating the optimal power-allocation policies to enhance transmission efficiency for wireless communications with multiple antennas and energy harvesting in a fading environment.

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A. Related Work

For energy harvesting under a fading channel, unlike the traditional battery-powered systems, energy is a random process that varies in time. On wireless communications, transmitter often sends packages to overcome the channel attenuation more rapidly than wired communications. The time between starting transmitting package i to package $i + 1$ is called epoch i. Often, the length of an epoch, which is called the time duration of the epoch, is written in specification. There has been recent research effort on understanding data transmission in this kind of systems, e.g., the investigation of power-allocation policies [3]–[8], medium access control protocols [9], adaptive opportunistic routing protocols [10], [11], network throughput of a mobile ad hoc network powered by energy harvesting [12], energy management in wireless sensor networks [7], [13], *etc*. In [3], data transmission with energy harvesting sensors is considered, and the optimal recursive policy for controlling admissions into the data buffer is derived using a dynamic programming framework. Dynamic programming can offer an optimal closed-loop control, but it needs to store a family of offline policies, where a huge storage space is often needed. In [4], energy management policies that stabilize the data queue are proposed for single-user communication and under a linear approximation. Some delay optimality properties are derived. In [5] and references therein, optimization approaches are considered to obtain the maximum throughput over additive white Gaussian noise (AWGN) and fading channels. Successively, in [6], optimal energy allocation to maximize the throughput is studied for energy harvesting systems in a time-constrained slotted setting. The same objective function is investigated in [5] and [6] using offline machinery and water-filling approach, which come directly from the Karush–Kuhn–Tucker (KKT) conditions [14] of the target problem. In [15], the optimality of a variant of the back pressure algorithm using energy queues is discussed. In [16], energy harvesting transmitters with batteries of finite capacity are considered, and the problem of throughput maximization is solved offline in a (nonfading) static channel. Recently, Gong *et al.* [17] has investigated the issue of optimal power allocation for energy harvesting and power grid

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coexisting wireless communication systems and has proposed the related methods.

The research on how to utilize the harvested energy efficiently in the MIMO system has just begun to appear. In [18], the precoding strategy along N independent channel use that maximizes the mutual information in linear vector Gaussian channels for arbitrarily distributed inputs is studied under noncausal knowledge of the harvested energy over time. In [19], the impact of deploying an energy harvesting multi-antenna cooperative jammer on the secrecy rate of a MIMO wiretap channel, with the remaining nodes being fixed power, is investigated.

B. Our Work

In our recently published paper [20], we have proposed an efficient geometric approach (GWF) to solve water-filling problems. The proposed GWF, as a functional block, was recursively used to solve the power allocation problem with energy harvesting under both fading and single-input–single-output (SISO) channels, i.e., single-antenna cases, in [21] and [22]. In this paper, we extend the algorithms in [21] to the MIMO system to solve the problems with energy harvesting causal constraints. Note that [21] cannot directly be used to solve the MIMO cases. Furthermore, it did not consider the related problems under a hybrid system with the coexistence of harvested energy and grid power. Due to the recursive feature and the repeated application of the GWF [20], the proposed algorithms are referred to as recursive GWF for MIMO (RGWFM) for throughput maximization and RGWFMn/RGWFMt for transmission completion time minimization problems. They can handle more general cases of the multiple antennas than the cases of the single antenna [21].

Compared with the existing results, the proposed algorithms own two significant and distinguished features: 1) It provides the exact optimal solution via finite computation with cubic polynomial computational complexity, and 2) its optimality is strictly proven. Due to the usage of recursion, the solutions to a family of the maximum throughput problems for any of the subprocesses starting from epoch 1 to epoch k, for $k = 1, \ldots, K$, can be obtained (where K is the index of the last epoch in the process).

Major Contribution in This Paper: Using our earlier geometric water-filling theory, the maximum throughput and the minimum transmission completion time problems for energy harvesting with multiple antennas are solved with optimal and exact solutions. The throughput maximization problem is further extended to a hybrid system, including grid power and optimal solutions being provided. To the best of the authors' knowledge, no existing algorithms reported in the open literature could provide exact solutions to the target problem.

Key notations that are used in this paper are as follows. $|\mathbf{A}|$ and $\mathrm{Tr}\,(\mathbf{A})$ give the determinant and the trace of a square matrix $\mathbf{A}$, respectively; $E[X]$ is the expectation of the random variable X; and the capital symbol $\mathbf{I}$ for a matrix denotes the identity matrix with the corresponding size. A square matrix $\mathbf{B} \succeq 0$ means that $\mathbf{B}$ is a positive semi-definite matrix. Further, for arbitrary two positive semi-definite matrices $\mathbf{B}$ and $\mathbf{C}$, the expression $\mathbf{B} \succeq \mathbf{C}$ means the difference of $\mathbf{B} - \mathbf{C}$ is a positive semi-definite matrix. In addition, for any complex matrix, its superscripts † and T denote the conjugate transpose and the transpose of the matrix, respectively.

In the remainder of this paper, the system model and the problems are presented in Section II. In Section III, the proposed RGWFM to maximize the throughput is discussed, and its optimality is proven by Proposition 1. In Sections IV, the proposed RGWFMn and RGWFMt algorithms are investigated to minimize the discrete and the continuous transmission completion time, respectively. Optimality of RGWFMn is proven by Proposition 2. Since proof of optimality of RGWFMt is similar to that of RGWFMn, it is omitted. In Section V, the throughput maximization problem in a hybrid system with the coexistence of energy harvesting and grid power is discussed. Optimality of its algorithm, i.e., RGWFMH, is proven by Proposition 3. In Section VI, numerical examples are presented. Section VII concludes this paper. For clarity, some notations or symbols are listed in Table I.

TABLE I
LIST OF VARIABLES AND ABBREVIATIONS

Key variables & abbreviations	Representations or interpretations
k^*	water level step (highest step under water)
E	expectation on probability
K	total number of epoches
$E_{in}(i)$	harvested energy at epoch i
$E_{(G,\text{total})}$	energy from power grid
$\mathbf{x^i}$	column vector, signal transmitted by the ith epoch,
$\mathbf{S_i}$	$E[\mathbf{x^i}\mathbf{x^i}^\dagger]$, a matrix
$\mathrm{Tr}(S_i)$	trace to represent power used at epoch
Equation (4)	complex maximum throughput problem which cannot directly be solved
RGWFM	algorithm for real maximum throughput problem (5)
RGWFMH	algorithm for hybrid or coexisting system with power grid (26)
RGWFMn	algorithm for minimum discrete completion time problem (17)
RGWFMt	algorithm for minimum continuous completion time problem (23)

II. SYSTEM MODEL AND PROPOSED PROBLEMS

Here, we introduce the system model and the transmission throughput maximization problem with energy harvesting in a fading channel. For convenience and without loss of generality, the process is assumed a discrete time process.

Fig. 1 shows the temporal structure of the epochs and energy arrivals, where it is assumed that a transmitter has an antenna and a receiver has an antenna, to depict the time period from $(0, T]$, including K epochs. We use L_i and a_i to denote the time duration and the fading gain of the ith epoch, respectively, where $\sum_{i=1}^{K} L_i = T$. $E_{\mathrm{in}}(1)$ is assumed the stored energy, which is observed at the beginning of the first epoch, as the initial available energy. At the beginning of the ith epoch ($i > 1$), the harvested energy is denoted by $E_{\mathrm{in}}(i)$. It can be used for transmitting information in the ith epoch and the following epochs. It is seen that $E_{\mathrm{in}}(i) \geq 0$. Hence, $E_{\mathrm{in}}(i) > 0$ accounts for a true energy arrival, whereas $E_{\mathrm{in}}(i) = 0$ accounts for no energy harvesting happening.

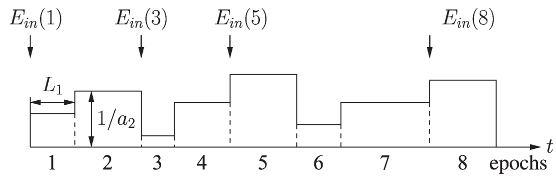

Fig. 1. Epochs and energy arrivals. $K = 8$ epochs in $(0, T]$.

For a MIMO-enhanced channel, assume that there are one receiver with N_r antennas and one transmitter or user, which is equipped with N_t antennas. The received signal at the ith epoch, i.e., $\mathbf{y}_i \in \mathbb{C}^{N_r \times 1}$, at the receiver is described as

$$\mathbf{y}_i = \mathbf{H}_i^\dagger \mathbf{x}^i + \mathbf{Z}, \quad \mathbf{H}_i \in \mathbb{C}^{N_t \times N_r}, \quad i = 1, 2, \ldots, K \tag{1}$$

where $\mathbf{H}_i$ is the channel gain matrix; $\mathbf{x}^i \in \mathbb{C}^{N_t \times 1}$ is the complex input signal vector transmitted at the ith epoch and is assumed a Gaussian random vector, having zero mean for any i; and $\{\mathbf{x}^i\}_{i=1}^K$ are independent on the meaning of probability theory. The noise term $\mathbf{Z} \in \mathbb{C}^{N_r \times 1}$ is an additive Gaussian noise random vector, i.e., $\mathbf{Z} \sim \mathbb{N}(0, \mathbf{I})$. The channel inputs $\{\mathbf{x}^i\}_{i=1}^K$ and $\mathbf{Z}$ are also assumed mutually independent. Furthermore, the covariance matrix of the transmit power at the ith epoch can be expressed as

$$\mathbf{S}_i \triangleq E\left[\mathbf{x}^i (\mathbf{x}^i)^\dagger\right], \qquad i = 1, 2, \ldots, K. \tag{2}$$

The total transmit power for the ith epoch is therefore $\mathrm{Tr}(\mathbf{S}_i)$. Note that $\mathbf{S}_i, \forall i$, is positively semi-definite, i.e., $\mathbf{S}_i \succeq 0$.

The objective is to maximize the number of bits transmitted by the deadline T, i.e., within the K epochs. Thus, the proposed problem is

$$\begin{aligned} \min_{\{\mathbf{S}_k\}_{k=1}^K} \quad & -\sum_{i=1}^K \frac{L_i}{2} \log \left| \mathbf{I} + \mathbf{H}_i^\dagger \mathbf{S}_i \mathbf{H}_i \right| \\ \text{Subject to} \quad & \mathbf{S}_i \succeq 0 \quad \forall i \\ & \sum_{k=1}^l L_i \mathrm{Tr}(\mathbf{S}_k) \le \sum_{k=1}^l E_{\text{in}}(k) \\ \text{for} \quad & l = 1, \ldots, K. \end{aligned} \tag{3}$$

Furthermore, for unifying parameter notation, through a change of variables, we can obtain an equivalent problem as follows:

$$\begin{aligned} \min_{\{\mathbf{S}_i\}_{i=1}^K} \quad & -\sum_{i=1}^K w_i \log \left| \mathbf{I} + \mathbf{H}_i^\dagger \mathbf{S}_i \mathbf{H}_i \right| \\ \text{subject to} \quad & \mathbf{S}_i \succeq 0 \quad \forall i \\ & \sum_{k=1}^l \mathrm{Tr}(\mathbf{S}_k) \le \sum_{k=1}^l E_{\text{in}}(k) \quad \forall l \end{aligned} \tag{4}$$

where $w_i \leftarrow L_i/2, H_i \leftarrow H_i/\sqrt{L_i}$, and $\mathbf{S}_i \leftarrow L_i \mathbf{S}_i$. Note that the symbol $\leftarrow$ is the assignment operator. Since the objective function of problem (4) with constraints has complex matrices $\mathbf{S}_i$ as the optimized variables, and the existing optimization theory and method only solve the real space problems, we need to transform problem (4) to an equivalent real-form problem, which is stated as follows:

$$\begin{aligned} \min_{\{s_i\}_{i=1}^{N_t \times K}} \quad & -\sum_{i=1}^{N_t \times K} w_{\left[\frac{i-1}{N_t}\right]+1} \log(1 + a_i s_i) \\ \text{subject to} \quad & 0 \le s_i \quad \forall i \\ & \sum_{i=1}^{N_t \times l} s_i \le \sum_{i=1}^l E_{\text{in}}(i) \quad \forall l \end{aligned} \tag{5}$$

where [ ] denotes the integral part of a real number. For simplicity, we write $\{w_{[(i-1)/N_t]+1}\}$ as $\{w_i\}$. The transformation for the mentioned equivalence between problem (4) and problem (5) is stated as follows. For problem (4), since

$$w_i \log \left| \mathbf{I} + \mathbf{H}_i^\dagger \mathbf{S}_i \mathbf{H}_i \right| = w_i \log \left| \mathbf{I} + \mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{S}_i \right| \qquad \forall i \tag{6}$$

there is the eigenvalue decomposition such that $\mathbf{H}_i \mathbf{H}_i^\dagger$ can be transformed into a diagonal matrix. This procedure is that there exists a unitary matrix $\mathbf{U}_i$ such that

$$\begin{aligned} \Lambda_i &= \mathbf{U}_i \mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{U}_i^\dagger \\ &= \begin{pmatrix} a_{(i-1)\times N_t + 1} & & \\ & \ddots & \\ & & a_{(i-1)\times N_t + N_t} \end{pmatrix} \end{aligned} \tag{7}$$

where Λ_i is a diagonal matrix and its diagonal element set $\{a_{(i-1)\times N_t + j}\}_{j=1}^{N_t}$, as a sequence, is monotonically decreasing. Thus

$$\begin{aligned} & w_i \log \left| \mathbf{I} + \mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{S}_i \right| \\ &= w_i \log \left(|\mathbf{U}_i| \left| \mathbf{I} + \mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{S}_i \right| \left| \mathbf{U}_i^\dagger \right| \right) \\ &= w_i \log \left| \mathbf{U}_i \mathbf{U}_i^\dagger + \Lambda_i \mathbf{S}_i' \right| \\ &= w_i \log \left| \mathbf{I} + \Lambda_i^{\frac{1}{2}} \mathbf{S}_i' \Lambda_i^{\frac{1}{2}} \right| \qquad \forall i \end{aligned} \tag{8}$$

where $\mathbf{S}_i' = \mathbf{U}_i \mathbf{S}_i \mathbf{U}_i^\dagger$ for simplifying notation. Furthermore, problem (4) is equivalent to the following problem:

$$\begin{aligned} \min_{\{\mathbf{S}_i'\}_{i=1}^K} \quad & -\sum_{i=1}^K w_i \log \left| \mathbf{I} + \Lambda_i^{\frac{1}{2}} \mathbf{S}_i' \Lambda_i^{\frac{1}{2}} \right| \\ \text{subject to} \quad & \mathbf{S}_i' \succeq 0 \qquad \forall i \\ & \sum_{k=1}^l \mathrm{Tr}\left(\mathbf{S}_k'\right) \le \sum_{k=1}^l E_{\text{in}}(k) \qquad \forall l. \end{aligned} \tag{9}$$

From the well-known Hadamard's inequality on positive definite matrices and some matrix operations, problem (9) is equivalent to problem (5). Note that since problem (4) is equivalent to problem (9) from the shown derivation, therefore, problem (4) is equivalent to problem (5). As a result, we only need to compute the solution to problem (5), and the solution is denoted by $\{s_i^*\}$. Successively, we can obtain

$$\mathbf{U}_i^\dagger \begin{pmatrix} s^*_{(i-1)\times N_t+1} & & \\ & \ddots & \\ & & s^*_{(i-1)\times N_t+N_t} \end{pmatrix} \mathbf{U}_i \quad \forall i \tag{10}$$

as the solution to problem (4). Note that the equivalence between two optimization problems can be guaranteed only by their having the same optimal value. Further, it need not require their objective values to be the same for any independent variable.

To find the solution to problem (5), the conventional water-filling approach usually starts from the KKT conditions of the problem and tries to solve a system of equations and inequalities in many optimization variables $\{s_i\}$ and the dual variables. Unlike the conventional approach, our proposed algorithm directly and efficiently solves the target problem by recursion and repeated application of our earlier proposed GWF. Due to our constructive solution, it also solves the KKT conditions.

III. RGWFM—Recursive GWF for Multiple Antennas

Here, we propose a novel approach to solve problem (5) using our proposed GWF approach.

Since the proposed RGWFM is based on a generalized algorithm of GWF [20], this generalized GWF is still termed as GWF that is concisely introduced as follows. GWF can be regarded as a mapping from the point of parameters $\{L', K', \{w_i\}_{i=L'}^{K'}, \{a_i\}_{i=L'}^{K'}, P\}$ to the solution $\{s_i\}_{i=L'}^{K'}$ and the important water level step index: k^* (which is defined as the highest channel/level index under water), where $\{a_i\}$ and $\{w_i\}$ are the fading gain vector and the weight vector, respectively [20]; L' and K' are two positive integers, to denote respectively the index of the starting channel and the ending channel of a set of channels (for power allocation) sorted according to their channel gains, i.e., $L' \leq K'$; and $K' - L' + 1$ is the total number of channels. Often, L' is assigned to be 1. That is, it can be written as the following formal expression:

$$\left\{\{s_i\}_{i=L'}^{K'}, k^*\right\} = \text{GWF}\left(L', K', \{w_i\}_{i=L'}^{K'}, \{a_i\}_{i=L'}^{K'}, P\right). \tag{11}$$

Thus, if $L' = 1$ and $K' = K$, this GWF is regressed into the original GWF. Since we often use the first part, i.e., $\{s_i\}_{i=L'}^{K'}$ from GWF, we also write

$$\{s_i\}_{i=L'}^{K'} = \text{GWF}\left(L', K', \{w_i\}_{i=L'}^{K'}, \{a_i\}_{i=L'}^{K'}, P\right). \tag{12}$$

Note that, for concision and without confusion from context, we may write the right-hand side of the expression as GWF(L', K') to emphasize time stages from L' to K'. Furthermore, the detailed definition, discussion, and optimality proof of GWF can be referred to in [20].

Through the mechanism of recursion, the solution $\{s_i^*\}_{i=1}^{KN_t}$ is obtained as RGWFM(K) within finite loops. Note that in Line 8, we used a summation. If the lower limit of the summation index is greater than the upper limit, the result of this summation is defined as zero.

Algorithm 1 Pseudocode of RGWFM

1. Initialize: $L = 1, N_t, K, P = E_{\text{in}}(1), \{w_k\}_{k=1}^{N_t}, \{a_k\}_{k=1}^{N_t}$;
2. Output the result for epoch 1:
   RGWFM(1) $=$ GWF$(1, N_t)|_I = \{s_k^*\}_{k=1}^{N_t}$.
3. For $L = 2 : 1 : K$,
4. Input: $\{E_{\text{in}}(L), \{w_{(L-1)N_t+j}, a_{(L-1)N_t+j}\}_{j=1}^{N_t}\}$;
5. $\{s'_k\}_{k=1}^{(L-1)N_t} =$ RGWFM$(L-1)$;
6. For $n = L : -1 : 1$,
7. $W = \{w_j\}_{j=(n-1)N_t+1}^{LN_t}$; $A = \{a_j\}_{j=(n-1)N_t+1}^{LN_t}$;
8. $S_T = \sum_{j=(n-1)N_t+1}^{(L-1)N_t} s'_j + E_{\text{in}}(L)$;
9. $\{\{s_k^*\}_{k=(n-1)N_t+1}^{LN_t}, k^*\}$
   $=$ GWF$((n-1)N_t + 1, LN_t, W, A, S_T)$;
10. $k_{\text{e}}^* = \max\{k | s'_k > 0,\ 1 \leq k \leq (n-1)N_t\}$;
11. If $\frac{1}{a_{k^*}w_{k^*}} + \frac{s_{k^*}}{w_k^*} \geq \frac{1}{a_{k_{\text{e}}}w_{k_{\text{e}}}} + \frac{s'_{k_{\text{e}}}}{w_{k_{\text{e}}}}$,
12. output: RGWFM(L)
    $= \{s'_1, \ldots, s'_{(n-1)N_t}, s^*_{(n-1)N_t+1}, \ldots, s^*_{LN_t}\}$,
13. Move to next epoch, i.e., go to Line 16;
14. End If
15. End For
16. End For

The proposed algorithm eliminates the procedure to solve the nonlinear system from the KKT conditions in multiple variables and dual variables, provides exact solutions via finite computation steps, and offers helpful insights to the problem and the solutions. To guarantee optimality of RGWFM, we have the following proposition.

Proposition 1: RGWFM can compute the optimal exact solution to problem (5) within finite loops.

Proof of Proposition 1: From the algorithm RGWFM(K), there exists n_1, where $1 \leq n_1 \leq K - 1$, and $\{s_i^*\}_{i=1}^{n_1 N_t} = [\text{RGWFM}(K)]|_{\{1,\ldots,n_1 N_t\}} = \text{RGWFM}(n_1)$. Thus, there are the nonnegative Lagrange dual variables $\{\lambda_i\}_{i=1}^{n_1}$ and $\{\mu_j\}_{j=1}^{n_1 N_t}$ such that KKT conditions, of the restriction of the optimization problem RGWFM(K) to the epoch set $\{1, \ldots, n_1\}$, hold. This restriction means a subproblem, i.e.,

$$\begin{aligned} \min_{\{s_i\}_{i=1}^{n_1 N_t}} \quad & \sum_{i=1}^{n_1 N_t} -w_i \log(1 + a_i s_i) \\ \text{subject to} \quad & \sum_{i=1}^{lN_t} s_i \leq \sum_{i=1}^{l} E_{\text{in}}(i), \ \text{as } 1 \leq 1 \leq n_1 - 1 \\ & 0 \leq s_i \qquad \forall i \\ & \sum_{i=1}^{n_1 N_t} s_i = \sum_{i=1}^{n_1 N_t} s_i^*. \end{aligned} \tag{13}$$

Furthermore, $\{\lambda_i, \{\mu_j\}\}$ correspond to the ith sum power constraint and power nonnegativeness constraint, respectively. On the other hand, $\{s_i^*\}_{i=n_1N_t+1}^{KN_t} = \text{GWF}(n_1N_t+1, KN_t, \{w_j\}, \{a_j\}, \sum_{j=n_1N_t+1}^{KN_t} s_j^*)$. Thus, there are also the nonnegative Lagrange dual variables $\{\lambda_i\}_{i=n_1+1}^{K}$ and $\{\mu_j\}_{j=n_1N_t+1}^{KN_t}$ that satisfy the KKT conditions of the following subproblem:

$$\begin{aligned}\min_{\{s_i\}_{i=n_1N_t+1}^{KN_t}} \quad & -\sum_{i=n_1N_t+1}^{KN_t} w_i \log(1+a_is_i)\\ \text{subject to} \quad & 0 \le s_i \qquad \forall\, i\\ & \sum_{i=n_1N_t+1}^{KN_t} s_i = \sum_{i=n_1N_t+1}^{KN_t} s_i^*. \end{aligned} \tag{14}$$

Since $\text{GWF}(n_1N_t+1, KN_t)$ has one sum power constraint and specific finite loop operations, we can assign

$$\lambda_K = \frac{1}{\frac{1}{a_{k^*}+w_{k^*}} + \frac{s_{k^*}}{w_{k^*}}} \tag{15}$$

$$\lambda_{K-1} = \cdots = \lambda_{n_1+1} = 0 \tag{16}$$

with the fine k^*, as the minimum positive step index of the set: $\{n_1N_t+1,\ldots,KN_t\}$, where the adjective "fine" expresses that k^* can be used to clarify whether the allocated power to be positive or zero and to determine the water level. In addition, due to the characteristics of the loop transition from n_1N_t+1 to n_1N_t when carrying out RGWFM(n_1) and $\text{GWF}(n_1N_t+1, KN_t, \{w_j\}, \{a_j\}, \sum_{j=n_1N_t+1}^{KN_t} s_j^*)$, and the points mentioned earlier, it is seen that the feasible solution of $\{s_i^*\}_{i=1}^{KN_t}$, computed by RGWFM(K), is indeed the optimal solution to (5) due to the fact that $\{\{\lambda_i\}_{i=1}^{K}, \{\mu_j\}_{j=1}^{KN_t}\}$ are the dual variables; they, together with $\{s_i^*\}_{i=1}^{KN_t}$, satisfy the KKT conditions of (5), and the qualification of (5) holds.

Therefore, Proposition 1 is proved.

IV. Transmission Completion Time Minimization

Earlier, RGWFM was discussed as a recursive water-filling to efficiently solve the throughput maximization problem. Here, RGWFM is used to solve the transmission completion time minimization problem.

Now, assume that the transmitter has B bits to be transmitted to the receiver. Our objective now is to minimize the time required to transmit these B bits. This problem is called the transmission completion time minimization problem. In [16] and [23], this problem is formulated and solved for an energy harvesting system in a nonfading environment. In [5], the problem is discussed offline in a fading channel by offering a condition the solution should meet. Single-antenna problems are investigated in those works. In this paper, we use RGWFM to solve the target problem in a fading channel with multiple antennas applying the recursion feature of the computation.

The transmission completion time minimization is categorized into two classes. The first class assumes that the completion time is taken at the ends of the epochs as discrete time points. Since T and $\{L_i\}$ are given, this class of problems just finds the minimum index of the epochs for transmission. The second class of the problem assumes that the completion time is taken at a time point that is continuously located in the interval $[0, T]$, as a continuous straight segment.

A. *Discrete Transmission Completion Time Minimization*

The discrete transmission completion time minimization problem can be stated as follows: Assume N to be a positive integer and $N \le K$, noting that the two notations N and N_t stand for the different meanings that the former is the index of an epoch; however, the latter is the number of the antennas equipped by a user. The problem can be written as

$$\begin{aligned}\min_{\{\{s_i\}_{i=1}^{NN_t}, N\}} \quad & N\\ \text{subject to} \quad & \sum_{i=1}^{NN_t} w_i \log(1+a_is_i) = B\\ & 0 \le s_i \qquad \forall\, i\\ & \sum_{i=1}^{lN_t} s_i \le \sum_{i=1}^{l} E_{\text{in}}(i),\ l = 1\ldots, K. \end{aligned} \tag{17}$$

We use the RGWFM machinery to design a recursive algorithm to solve (17), referred to as RGWFMn. The steps of the RGWFMn is presented in the algorithm description shown in the following.

Algorithm 2 Algorithm RGWFMn, Based on RGWFM

1. Initialize: $L=1, N_t, K, E_{\text{in}}(1), \{w_k\}_{k=1}^{N_t}, \{a_k\}_{k=1}^{N_t}, B$;
2. Output the result for epoch 1:
   $\text{RGWFM}(1) = \text{GWF}(1, N_t, \{w_i\}, \{a_i\}, E_{\text{in}}(1))|_I$
   $= \{s_k^*\}_{k=1}^{N_t}$;
3. If $\sum_{i=1}^{N_t} w_i \log(1+a_is_i^*) \ge B$,
4. $\{s_i^*\}_{i=1}^{N_t} = \text{GWF}(1, N_t, \{w_i\}, \{a_i\}, P_1)$,
   where $P_1 = \sum_{i=1}^{k_1^*}[\frac{1}{a_{k_1^*}}(\frac{2^{\frac{B}{w_1}}}{\prod_{i=1}^{k_1^*-1}(\frac{a_i}{a_{k_1^*}})})^{\frac{1}{k_1^*}} - \frac{1}{a_i}]$,
   $k_1^* = \max\{k|\prod_{i=1}^{k-1}(\frac{a_i}{a_k}) < 2^{\frac{B}{w_1}}, 1 \le k \le N_t\}$,
   $N^* = 1$, and then exit the algorithm;
5. End If
6. For $L = 2:1:K$,
7. Input: $\{E_{\text{in}}(L), \{w_{(L-1)N_t+j}, a_{(L-1)N_t+j}\}_{j=1}^{N_t}\}$;
8. $\{s_k'\}_{k=1}^{(L-1)N_t} = \text{RGWFM}(L-1)$;
9. For $n = L:-1:1$,
10. $W = \{w_j\}_{j=(n-1)N_t+1}^{LN_t}; A = \{a_j\}_{j=(n-1)N_t+1}^{LN_t}$;
11. $S_T = \sum_{j=(n-1)N_t+1}^{(L-1)N_t} s_j' + E_{\text{in}}(L)$;
12. $\{\{s_k^*\}_{k=(n-1)N_t+1}^{LN_t}, k^*\}$
    $= \text{GWF}((n-1)N_t+1, LN_t, W, A, S_T)$;
13. $k_{\text{e}}^* = \max\{k|s_k' > 0,\ 1 \le k \le (n-1)N_t\}$;
14. If $\frac{1}{a_{k^*}w_{k^*}} + \frac{s_{k^*}}{w_k^*} \ge \frac{1}{a_{k_{\text{e}}}w_{k_{\text{e}}}} + \frac{s'_{k_{\text{e}}}}{w_{k_{\text{e}}}}$,
15. $\text{RGWFM}(L)$
    $= \{s_1', \ldots, s'_{(n-1)N_t}, s^*_{(n-1)N_t+1}, \ldots, s^*_{LN_t}\}$,
16. If $\sum_{i=1}^{(n-1)N_t} w_i \log(1+a_is_i')$
    $+ \sum_{i=(n-1)N_t+1}^{LN_t} w_i \log(1+a_is_i^*) \ge B$,

17. $B_1 = B - \sum_{i=1}^{(n-1)N_t} w_i \log(1 + a_i s_i')$ $- \sum_{i=(n-1)N_t+1}^{(L-1)N_t} w_i \log(1 + a_i s_i^*)$,
18. $\{s_i^*\}_{i=(L-1)N_t+1}^{LN_t}$ can be obtained, similar to Line 4,
19. RGWFM(L) $= \{s_i'\}_{i=1}^{(n-1)N_t} \cup \{s_j^*\}_{j=(n-1)N_t+1}^{LN_t}$,
20. $N^* = L$, and then exit the algorithm;
21. End If
22. Move to next epoch, i.e., go to Line 25;
23. End If
24. End For
25. End For

Compared with the steps of RGWFM, in the first line of the algorithm, RGWFMn introduces B as a parameter while others being kept unchanged. In Line 6, RGWFMn sequentially processes from the second epoch to the Kth epoch to output the optimal value N^* and its optimal solution, i.e., RGWFM(N^*), with the target rate B bits. Similarly, the inner "For" loop updates power levels for the current processing epoch (L) and its previous $(L - n + 1)$ epochs to form a processing window. The GWF algorithm is also applied to this window to find a common water level. Note that a new "If" clause is inserted into the outer level "If" clause (for the water level nondecreasing condition check). The function of this inner "If" clause is to check whether and how the transmitted bits reach B. Therefore, it is the normal exit of the algorithm (lines 19–20). For convenience, the condition of this new "If" clause is called the criterion of RGWFMn. This is also due to the importance of the criterion in the following proposition. Note that the truth of Line 4 is easy to test. In addition, Line 4 of RGWFMn provided a solution, but this solution is not unique. However, this proposed solution uses the least power to guarantee the rate requirement $(= B)$.

To guarantee optimality of RGWFMn, the proposition is stated as follows.

Proposition 2: If there does not exist L such that the criterion in RGWFMn

$$\sum_{i=1}^{(n-1)N_t} w_i \log(1+a_i s_i') + \sum_{i=(n-1)N_t+1}^{LN_t} w_i \log(1+a_i s_i^*) \geq B \tag{18}$$

holds, where the symbols in (18) keep the same meaning as those in the statement of RGWFMn, then there is no solution to problem (17). If the criterion holds, then the obtained N^* is the optimal value, and the $\{\text{RGWFM}(N^*), N^*\}$ is the exact optimal solution.

Proof of Proposition 2: For the given B, if there does not exist L and n such that the criterion in RGWFMn

$$\sum_{i=1}^{(n-1)N_t} w_i \log(1 + a_i s_i') + \sum_{i=(n-1)N_t+1}^{LN_t} w_i \log(1 + a_i s_i^*) \geq B \tag{19}$$

holds, it implies that the optimal value of problem (5) is strictly less than B, corresponding to Proposition 1. Thus, the first constraint in problem (17) never holds. Then, there is no solution to problem (17).

Then, assume that there exist N^* and RGWFM(N^*) such that

$$\sum_{i=1}^{(n-1)N_t} w_i \log(1 + a_i s_i') + \sum_{i=(n-1)N_t+1}^{N^* N_t} w_i \log(1 + a_i s_i^*) \geq B \tag{20}$$

where

$$\text{RGWFM}(N^*) = \left\{ s_1', \ldots, s_{(n-)N_t}', s_{(n-1)N_t+1}^*, \ldots, s_{N^* N_t}^* \right\}. \tag{21}$$

According to the obtained N^* from the RGWFMn algorithm, the optimal value of the problem

$$\begin{aligned} \min_{\{s_i\}_{i=1}^{NN_t}} \quad & \sum_{i=1}^{NN_t} -w_i \log(1 + a_i s_i) \\ \text{subject to} \quad & 0 \leq s_i \quad \forall i \\ & \sum_{i=1}^{lN_t} s_i \leq \sum_{i=1}^{lN_t} E_{\text{in}}(i) \text{ for } l = 1, \ldots, N \end{aligned} \tag{22}$$

is less than B, where RGWFM(N) is the optimal solution to this problem, for $N = 1, \ldots, N^* - 1$. Hence, the optimal value of problem (17) is not less than N^*. Stemming from the statement of RGWFMn, RGWFM(N^*) is a feasible solution to problem (17), and N^* is the evaluated objective value of problem (17) at RGWFM(N^*). Thus, N^* is a feasible value. Together with the mentioned fact that the optimal value of problem (17) is not less than N^*, as a result, N^* is the optimal value, and $\{\text{RGWFM}(N^*), N^*\}$ is the exact optimal solution to problem (17).

Therefore, Proposition 2 is proved.

Note that, for the first constraint, in (17), if the power values make the weighted sum rate greater than the target B, we may reduce the power values to make the weighted sum rate equal to B. The reduced power is just a feasible solution and the sum power becomes less. This result comes from continuity of the throughput or rate constraint function and the rest of the constraints. Therefore, the problem that substitutes the inequality of "$\geq$" for the equality of "$=$" in the first constraint has the same optimal solution set.

B. Continuous Transmission Completion Time Minimization

The continuous transmission completion time minimization problem can be stated as follows: Assume t to be a real number and N to denote the index variable of the sequence, consisting of the ends for the progressive epochs $\mathbb{Z}$ denotes the set of integers, and then, the corresponding objective function is shown as in (23), shown below. Note the previously mentioned $w_i = L_i/2\,\forall i$.

$$\begin{aligned}
\min_{\{\{s_i\}_{i=1}^{NN_t},t\}} \quad & t \\
\text{subject to} \quad & 1 \le N \le K \text{ and } N \in \mathbb{Z} \\
& N_1(t) = \max\left\{N \Big| \sum_{k=1}^{N} L_k \le t\right\} \\
& \sum_{l=1}^{N_1}\sum_{j=1}^{N_t} w_{(l-1)N_t+j} \\
& \quad \cdot \log\left(1 + a_{(l-1)N_t+j}\cdot s_{(l-1)N_t+j}\right) \\
& \quad + \left(\frac{t}{2} - \sum_{k=1}^{N_1} w_k\right) \\
& \quad \cdot \sum_{j=1}^{N_t} \log\left(1 + a_{(N_1-1)N_t+j} s_{(N_1-1)N_t+j}\right) = B \\
& 0 \le s_i \qquad \forall i;\ 0 \le t \le T \\
& \sum_{i=1}^{lN_t} s_i \le \sum_{i=1}^{l} E_{\text{in}}(i),\ l = 1,\ldots,K.
\end{aligned} \tag{23}$$

If Lebesgue–Stieltjes integration [24] is used for problem (23), it can make the expression concise. The presented method is used to avoid introducing more abstract mathematical expressions. We use the proposed RGWFMn to design a recursive algorithm to solve the continuous transmission completion time minimization problem (23), which is referred to as RGWFMt. The steps of the RGWFMt are stated as follows: RGWFMt only replaces Line 18, and the $N^* = L$ of Line 20 of RGWFMn with the statement

$$\Delta t^* = \frac{2B_1}{\sum_{j=1}^{N_t} \log\left(1 + a_{(N^*-1)N_t+j} s_{(N^*-1)N_t+j}\right)} \tag{24}$$

and the statements

$$N^* = L, t^* = \Delta t^* + \sum_{k=1}^{N^*-1} L_k \tag{25}$$

respectively. The optimality proof of RGWFMt can be proven, similarly to that of RGWFMn: Proposition 2. Therefore, its proof is ignored in this paper.

V. Extension to a Hybrid System Coexisting With Grid Power

Since energy harvesting depends on natural condition and is a random process, the energy from the power grid is often added as a supplementary source. The corresponding maximum throughput problem of such a hybrid system can be stated as

$$\begin{aligned}
\min_{\{\mathbf{S}_i,\mathbf{S}_{G,i}\}_{i=1}^K} \quad & -\sum_{i=1}^{K} w_i \log\left|\mathbf{I} + \mathbf{H}_i^\dagger(\mathbf{S}_i + \mathbf{S}_{G,i})\mathbf{H}_i\right| \\
\text{subject to} \quad & \mathbf{S}_i \succeq 0 \qquad \forall i \\
& \mathbf{S}_{G,i} \succeq 0 \qquad \forall i \\
& \sum_{k=1}^{l} \text{Tr}(\mathbf{S}_k) \le \sum_{k=1}^{l} E_{\text{in}}(k) \qquad \forall l \\
& \sum_{k=1}^{K} \text{Tr}(\mathbf{S}_{G,k}) \le E_{(G,\text{total})}
\end{aligned} \tag{26}$$

where $\mathbf{S}_{G,k} \forall k$ is the power from power grid, and $E_{(G,\text{total})}$ is the total energy from power grid. To easily understand the essence and the proof of this extension and avoid more subscripts being used, without loss of generality, let $N_t = 1$ here.

For problem (26), We add a statement

$$\{s_{G,k}^*\}_{k=1}^K = \text{GWF}\left(1, K, W, \frac{a_k}{1 + a_k s_{H,k}^*}, E_{(G,\text{total})}\right)|_I \tag{27}$$

at the end of RGWFM for grid power allocation. We refer to this as RGWFMH. RGWFMH implies that, for this hybrid system, harvested energy is first allocated as in the algorithm RGWFM. The results from RGWFM define the new water tank bottom. The total available grid power is then allocated again with the water-filling algorithm.

The optimality of RGWFMH is stated and proven by the following proposition.

Proposition 3: RGWFMH can compute the optimal exact solution to the problem (26) within finite loops.

Proof of Proposition 3: Similar to the transformation from (4) to (5), problem (26) has its real representation or problem. For clarity, $\{s_{H,i}\}$ denotes the power from energy harvesting and $\{s_{G,i}\}$ from the power grid.

According to Proposition 1, for the real form of problem (26) under $E_{(G,\text{total})} = 0$, there exist the optimal solution $\{s_{H,i}\}_{i=1}^K$ and the dual variables $\{\lambda_i, \mu_i\}_{i=1}^K$ that satisfy the following KKT conditions:

$$\begin{cases}
\frac{1}{\frac{1}{a_i w_i} + \frac{s_{H,i}}{w_i}} = \sum_{k=i}^K \lambda_k - \mu_i \qquad \forall i \\
\mu_i s_{H,i} = 0,\ s_{H,i} \ge 0,\ \mu_i \ge 0 \quad \forall i \\
\lambda_l\left(\sum_{k=1}^l s_{H,k} - \sum_{k=1}^l E_{\text{in}}(k)\right) = 0 \\
\sum_{k=1}^l s_{H,k} \le \sum_{k=1}^l E_{\text{in}}(k),\ \lambda_l \ge 0 \qquad \forall l.
\end{cases} \tag{28}$$

We define the index set Λ_1 as follows:

$$\Lambda_1 = \{i | s_{H,i} > 0\} = \{i_t | 1 \le i_1 < i_2 < \cdots < i_{t_1} \le K\}. \tag{29}$$

Expression (29) combined with RGWFM(K) implies that

$$\frac{1}{a_{i_k} w_{i_k}} + \frac{s_{H,i_k}}{w_{i_k}} \le \frac{1}{a_{i_{k+1}} w_{i_{k+1}}} + \frac{s_{H,i_{k+1}}}{w_{i_{k+1}}} \tag{30}$$

for any $\{i_k, i_{k+1}\} \subset \Lambda_1$. Note that optimal solution satisfies the mentioned inequalities (30), but only some variables that meet these inequalities cannot guarantee themselves to be (the part of) the optimal solution.

Let

$$\begin{aligned}\left\{\{s_{G,k}\}_{k=1}^{K}, k^*\right\} \\ = \text{GWF}\left(1, K, W, \left\{\frac{a_k}{1 + a_k S_{H,k}^*}\right\}, E_{(G,\text{total})}\right). \quad (31)\end{aligned}$$

It is seen that

$$\frac{1}{a_{i_t} w_{i_t}} + \frac{s_{H,i_t}}{w_{i_t}} + \frac{s_{G,i_t}}{w_{i_t}} = \frac{1}{a_{k^*} w_{k^*}} + \frac{s_{G,k^*}}{w_{k^*}} + \frac{s_{G,k^*}}{w_{k^*}} \quad (32)$$

for any $\{k^*, i_t\} \subset \Lambda_1$, where $1 \leq i_t \leq k^* \, \forall t$.

Let $\overline{\mu}_{i_t} = 0$ as $i_t \in \Lambda_1$; $\overline{\lambda} = 1/((1/a_{k^*} w_{k^*}) + (s_{H,k^*}/w_{k^*}) + (s_{G,k^*}/w_{k^*}))$. $\overline{\lambda}_{k^*} = \overline{\lambda}$, and $\overline{\lambda}_l = 0$ as $l \in \{1, 2, \ldots, K\} \setminus \{k^*\}$. In addition, $\overline{\mu}_j = (\overline{\lambda}_{k^*} - (1/(1/a_j w_j) + (s_{H,j}/w_j)))^+$ as $j \in \{1, 2, \ldots, K\} \setminus \Lambda_1$. Moreover, $\overline{\nu}_{i_t} = 0$ as $i_t \in \Lambda_1$ and $\overline{\nu}_j = (1/((1/a_j w_j) + (s_{H,j}/w_j) + (s_{G,j}/w_j)) - \overline{\lambda})^+$ as $j \in \{1, 2, \ldots, K\} \setminus \Lambda_1$.

Therefore, the $\{s_{H,i}, s_{G,i}\}_{i=1}^{K}$ aforementioned and the constructed dual variables $\{\overline{\lambda}_i, \overline{\mu}_i\}_{i=1}^{K}$ and $\{\overline{\lambda}, \overline{\nu}_1, \ldots, \overline{\nu}_K\}$ satisfy the KKT conditions of the real form of problem (26), the Lagrange function of which is

$$\begin{aligned}L&\left(\{s_{H,i}, s_{G,i}\}_{i=1}^{K}; \{\overline{\lambda}_i, \overline{\mu}_i\}_{i=1}^{K}, \{\overline{\lambda}, \overline{\nu}_1, \ldots, \overline{\nu}_K\}\right) \\ &= -\sum_{i=1}^{K} w_i \log\left(1 + a_i(s_{H,i} + s_{G,i})\right) - \sum_{i=1}^{K} \mu_i s_{H,i} \\ &\quad + \sum_{l=1}^{K} \lambda_l \left(\sum_{k=1}^{l} s_{H,k} - \sum_{k=1}^{l} E_{\text{in}}(k)\right) \\ &\quad - \sum_{i=1}^{K} \nu_i s_{G,i} + \lambda\left(\sum_{k=1}^{K} s_{G,k} - E_{(G,\text{total})}\right). \quad (33)\end{aligned}$$

In addition, by observation, the general constraint qualification of problem (26) holds. Then, $\{s_{H,i}, s_{G,i}\}_{i=1}^{K}$ computed by the proposed RGWFMH is the optimal solution to problem (26).

Therefore, Proposition 3 is proved.

As a remark, we should emphasize two points. First, RGWFMH is actually a block coordinate ascent algorithm (BCAA, to find maximum) ([14]). BCAA is only guaranteed to be infinite iterations to find an optimal solution to the proposed problem. However, we further exploit the structure of the proposed problem, and then, the proposed RGWFMH is designed. Further, RGWFMH just uses one time iteration and rapidly obtains the optimal solution. Second, we may treat $E_{(G,\text{total})}$ to be the harvest energy at the starting of the first epoch and can obtain the equivalent solution, but we cannot obtain or distinguish the optimal power allocation from the two different energy sources. With the implementation of RGWFMH, this problem can be solved efficiently.

VI. Numerical Examples and Computational Complexity

Here, we provide the numerical examples and computational complexity analysis. In the numerical examples, we one simple example to clearly illustrate the procedures of the proposed algorithms, and two more complicated examples to compare with the primal–dual interior point method (PD-IPM), which is now regarded as an efficient optimization algorithm with great promise (see [25] and references therein). In the computational complexity analysis, we discussed the computational complexity of the proposed algorithms and arrived at the conclusion of polynomial complexity (see [26]). Due to exploiting the structure of the proposed problems, the proposed algorithms show significant efficiency.

A. Numerical Examples

The proposed algorithms scan the epochs sequentially to obtain the optimal power allocation.

Example 1: We assume that there are three epochs, each with unit length ($L_i = 1,\ i = 1, 2, 3$) and the same unit weight ($w_i = 1/2,\ i = 1, 2, 3$). To clearly account for the procedures of the proposed algorithms, no power grid case is considered without loss of essence.

Let $N_t = N_r = 2$. At the beginning of each epoch, the energy is harvested as $E_{\text{in}}(i) = 2,\ i = 1, 2, 3$. The information required for transmission is $B = 3$ bits. Suppose the fading profile for the three epochs is

$$\begin{aligned}\mathbf{H}_1 &= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}, \quad \mathbf{H}_2 = \begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix} \\ \mathbf{H}_3 &= \sqrt{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}. \quad (34)\end{aligned}$$

First, RGWFM(1) outputs $\{s_1 = 1, s_2 = 1\}$, i.e., the allocated power sum is $\text{Tr}(\mathbf{S}_1) = 2$ to epoch 1, as shown in Fig. 2(a). The height of the blue (darker) stair bars denotes the reciprocals of the fading gains, i.e., the height of the steps for water-filling [20]. The allocated powers are illustrated by the height of the green (gray) bars. Along the axis of "Index of epoch," the index of epochs increases from left to right.

Then, the process moves to the second epoch. By applying GWF(2,2), it gives $\{s_3 = 1, s_4 = 1\}$. Now, check if the water level of epoch 2 is $1 + 1/2 = 1.5$ and if the water level for epoch 1 is $1 + 1 = 2$. The water-level nondecreasing condition is violated. The power-level adjustment procedure is triggered. By applying GWF to the first two epochs, we have $\text{GWF}(1, 2) = \{s_1 = 0.75, s_2 = 0.75; s_3 = 1.25, s_4 = 1.25\}$. With this power adjustment, the output for RGWFM(2) is then $\{s_1 = 0.75, s_2 = 0.75; s_3 = 1.25, s_4 = 1.25\}$, i.e., $\text{Tr}(\mathbf{S}_1) = 1.5$, and $\text{Tr}(\mathbf{S}_2) = 2.5$, as shown in Fig. 2(b).

Now, the process moves to epoch 3, and the output of $\text{GWF}(3, 3) = \{s_5 = 1, s_6 = 1\}$. The corresponding water level for epoch 3 is $1 + 1/4$, which is lower than that of the previous epoch (=1.75). Then, the power adjustment is triggered. After two power adjustment operations, the final output is $\{s_1 = 7/12, s_2 = 7/12; s_3 = 13/12, s_4 = 13/12;$

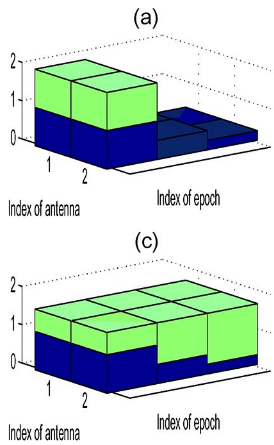

Fig. 2. Procedures to solve Example 2. (a) $\mathrm{Tr}(\mathbf{S}_1) = 2$. (b) $\mathrm{Tr}(\mathbf{S}_1) = 3/2$, $\mathrm{Tr}(\mathbf{S}_2) = 5/2$. (c) $\mathrm{Tr}(\mathbf{S}_1) = 14/12$, $\mathrm{Tr}(\mathbf{S}_2) = 26/12$, $\mathrm{Tr}(\mathbf{S}_3 = 8/3$. (d) $\mathrm{Tr}(\mathbf{S}_1) = 7/6 \doteq 1.17$, $\mathrm{Tr}(\mathbf{S}_2) = 26/12 \doteq 2.17$, $\mathrm{Tr}(\mathbf{S}_3) = 2\log(4 \cdot (12/19)^2) \doteq 1.35$.

$s_5 = 4/3, s_6 = 4/3\}$, which is the completed output for the optimal solution, i.e., $\mathrm{Tr}(\mathbf{S}_1) = 14/12, \mathrm{Tr}(\mathbf{S}_2) = 26/12$, and $\mathrm{Tr}(\mathbf{S}_3) = 8/3$, as shown in Fig. 2(c).

In this example, the channel states (fading gains) for the three epochs are continuously improving. Therefore, the harvested energy at the beginning of each epoch tries to flow to the later epochs, leading to the uniform water level of these three epochs. In addition, if $N_t = N_r = 1$, the power gains are reduced into $h_i = 2^{i-1}$, $i = 1, 2, 3$. It can be calculated that the maximum transmission throughput is $3\log(31/6)$. However, if $N_t = N_r = 2$, the maximum transmission throughput is $2\log(8 \cdot (19/12)^3)$. Thus, the throughput ratio of the MIMO case to the SISO case is $(2 \cdot \log(8 \cdot (19/12)^3)/3 \cdot \log(31/6)) \doteq 1.40$ by adding one more antenna at both the transmitter and the receiver.

Let us compute the solutions to two types of the minimum transmission completion time problems. For the final result over the entire process, the optimal solution is $\{\{s_1 = 7/12, s_2 = 7/12; s_3 = 13/12, s_4 = 13/12; s_5^* = \log(4 \cdot (12/19)^2), s_6^* = \log(4 \cdot (12/19)^2)\}, N^* = 3\}$, and the optimal value is $N^* = 3$, i.e., the allocated power is $\mathrm{Tr}(\mathbf{S}_3) = 2\log(4 \cdot (12/19)^2)$ to epoch 3, as shown in Fig. 2(d).

For continuous solution t, we obtain a different solution using RGWFMt, i.e., $\{s_1 = 7/12, s_2 = 7/12; s_3 = 13/12, s_4 = 13/12; s_5^* = 4/3, s_6^* = 4/3$, and

$$\begin{aligned} t^* &= 2 + \frac{2B_1}{\sum_{j=1}^{N_t} \log\left(1 + a_{(N^*-1)N_t+j} s_{(N^*-1)N_t+j}\right)} \\ &= 2 + \frac{\log\left(4 \cdot (12/19)^2\right)}{\log(1+4 \cdot 4/3)} \\ &\doteq 2.25. \end{aligned} \tag{35}$$

Through this simple example, the computation procedures and the effectiveness of the proposed algorithms are well demonstrated. The following general example further reveals the effectiveness of the proposed algorithms.

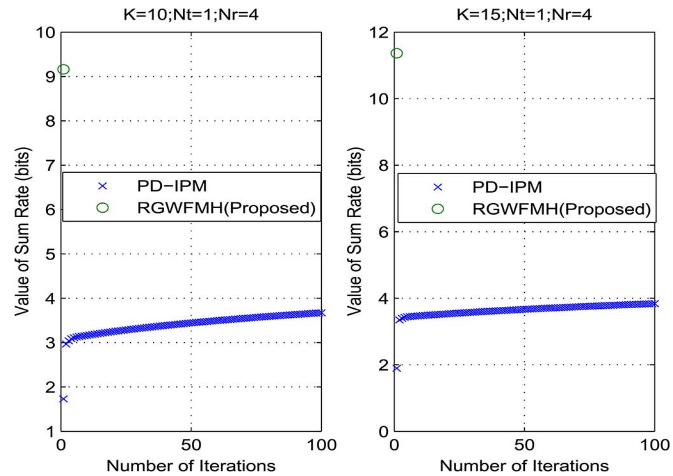

Fig. 3. Weighted sum rates (Unit: bits) of RGWFMH and PD-IPM, as K = 10 and 15.

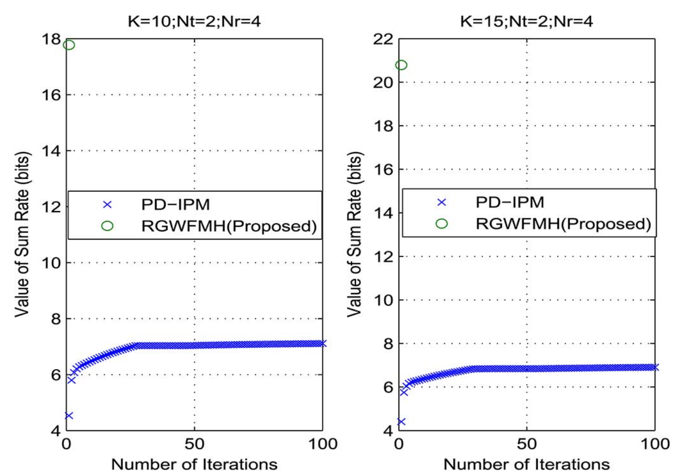

Fig. 4. Weighted sum rates (Unit: bits) of RGWFMH and PD-IPM, as K = 10 and 15.

Example 2: The well-known optimization algorithm, i.e., the PD-IPM, is chosen for comparison purpose due to its competitiveness in computing the solutions to the convex optimization problems. The proposed minimum transmission completion time problems are nonconvex mixed-integer optimization problems. As far as the authors' knowledge, there is no algorithm reported in the open literature that can compute the exact solutions to the target problems. As a result, we only focus on the throughput maximization problem.

Figs. 3–5 are used to show the difference between PD-IPM and RGWFMH for the maximum throughput problems, through some choices of the number of antennas (N_t) at the user or the number of epochs (K). In calculation, the number of antennas at base station (N_r) is set to be 4. Channel gains are generated randomly using random $N_r \times N_t$ matrices with each entry drawn independently from the standard Gaussian distribution. $\{E_{\mathrm{in}}(k)\}$ is the set of randomly chosen positive numbers. The sum power constraint of the power grid $E_{(G,\mathrm{total})}$ is 5. A group of different weights are also generated randomly. The chosen parameters aforementioned are assigned to both algorithms with the identical values for comparability. In these figures, the circle markers and the cross markers represent the results

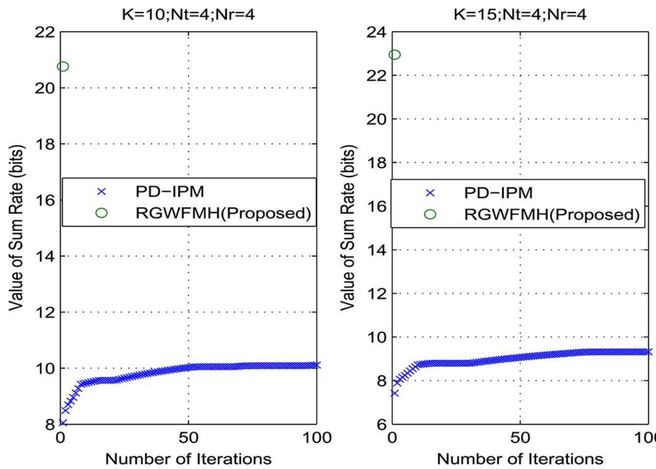

Fig. 5. Weighted sum-rates (Unit: bits) of RGWFMH and PD-IPM, as K = 10 and 15.

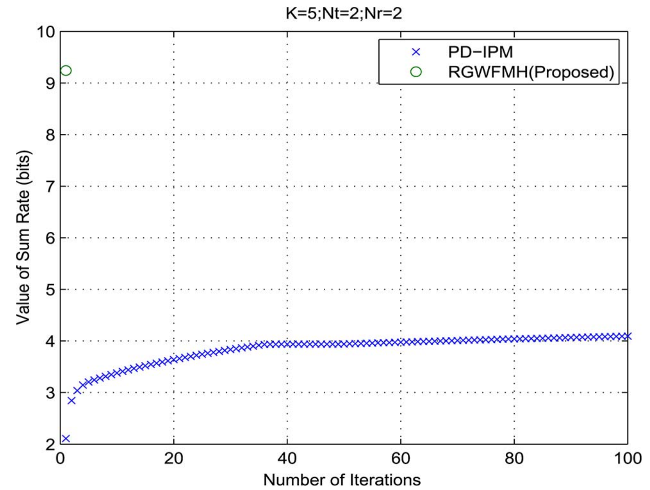

Fig. 6. Weighted sum-rates (Unit: bits) of RGWFMH and PD-IPM, as K = 5.

TABLE II
COMPARISON OF THE ACHIEVED THROUGHPUT ($N_r = 4$)

parameter	PD-IPM	RGWFMH	ratio
$K = 10, N_t = 1$	3.67	9.15	0.40
$K = 15, N_t = 1$	3.84	11.36	0.34
$K = 10, N_t = 2$	7.11	17.77	0.40
$K = 15, N_t = 2$	6.91	20.79	0.33
$K = 10, N_t = 4$	10.11	20.76	0.49
$K = 15, N_t = 4$	9.32	22.95	0.41

of the proposed RGWFMH and PD-IPM, respectively. For the proposed RGWFMH, since it uses recursion, no iteration is invoked. Therefore, the number of iterations of the circles maps to one iteration. The obtained throughput is summarized in the following table. The obtained throughput for PD-IPM is the result after 100 iterations.

With different parameters, the achieved throughput ratio of the PD-IPM to that of RGWFMH is in the range of 0.33 to 0.49 (see Table II). These results show that the proposed RGWFMH exhibits much better performance. It also shows that, as the number of the antennas increases, the throughput or the weighted sum rate increases.

Example 3: For ease to follow the simulation results, a deterministic example is given. The parameters are chosen as follows: Assume there are five epochs with weight factor vector $W = \{0.1633, 0.2132, 0.2282, 0.2035, 0.1918\}$, $E_{\text{in}}(i) = 6$, $\forall\, i$, and $E_{(G,\text{total})} = 5$, the channel gain matrices of the two-by-two antenna array are randomly generated as

$$H_1^{\dagger} = \begin{pmatrix} -0.2056 + 0.1700i & -0.3895 - 0.6354i \\ 0.2236 + 0.2518i & 1.5094 - 1.0604i \end{pmatrix}$$

$$H_2^{\dagger} = \begin{pmatrix} 0.3851 - 0.2639i & 1.6777 + 0.3762i \\ -0.1068 - 0.1593i & -0.3660 - 0.9417i \end{pmatrix}$$

$$H_3^{\dagger} = \begin{pmatrix} 0.2877 + 0.5690i & 0.5789 + 0.8900i \\ -0.2702 - 0.5321i & -0.2975 - 0.5033i \end{pmatrix}$$

$$H_4^{\dagger} = \begin{pmatrix} -0.2851 - 0.5181i & 0.3035 - 0.1812i \\ 0.1038 - 0.4797i & 0.4999 - 0.4366i \end{pmatrix}$$

$$H_5^{\dagger} = \begin{pmatrix} -0.7143 - 0.6832i & -0.1870 - 0.7028i \\ 0.2136 - 0.5346i & 0.2199 - 1.1445i \end{pmatrix}. \quad (36)$$

The optimal power allocation of the harvested energy is {4.6740, 6.1910, 6.4261, 5.1950, 7.5139}, the ith member of which corresponding to the ith epoch. Similarly, the optimal power allocation of the grid power is {0.6851, 0.8944, 0.9574, 0.8538, 1.6094}. The throughput is shown in Fig. 6. It is seen that the achieved throughput of the proposed algorithm is 9.2, but the corresponding value of PD-IPM is 4.1, which reflects an almost 1.2 times gain in the achieved throughput.

B. Computational Complexity Analysis

Since the proposed algorithms use the proposed real throughput mathematical representation by the unitary similarity matrix transformation, the proposed algorithms need the unitary similarity matrix transformation with computational complexity $K \times O(N_t^3)$ (refer to [26]) for (5). Second, to compute the optimal solution, RGWFMH, as a more general case than RGWFM, utilizes GWF $\sum_{L=1}^{K}(1 + L)L/2 + 1$ times; therefore, it needs $\sum_{L=1}^{K}\sum_{k=1}^{L}(8N_t k + 3) + 8N_t K + 3 = 4N_t K[K^2 + 3K + 8]/3 + 3K(K + 1)/2 + 3$, i.e., $N_t O(K^3)$ fundamental operations (refer to [20]). Because a valid algorithm needs to apply the proposed real throughput mathematical representation to avoid the differentiability problem from the several complex optimization variables; therefore, the comparison only focuses on the computational complexity led by the computation of the optimal solution. The complexity of RGWFMH is rather low $N_t O(K^3)$. For example, although we let PD-IPM use the proposed transform to obtain an equivalent real problem, however, for the ϵ solution, i.e., not the optimal solution, it still needs a polynomial computational complexity, i.e., $N_t^{3.5} O(K^{3.5}) \log(1/\epsilon)$ (refer to [25] and [27]). Hence, PD-IPM cannot guarantee to output the optimal solution by finite computation. Our method eliminates any linear search but output the exact optimal solution with finite computation.

Note the difference between iteration and recursion. The linear search is often computationally demanding. This is one weakness of PD-IPM. When the feasible set has the sharp boundary where optimal point(s) is located and the objective function is nonlinear, this weakness appears to be more remarkable.

Simply speaking, RGWFMH needs total $K \times O(N_t^3) + N_t O(K^3)$ basic operations to compute the exact solution, whereas PD-IPM needs total $K \times O(N_t^3) + N_t^{3.5} O(K^{3.5}) \log(1/\epsilon)$ basic operations to compute an ϵ solution.

VII. Conclusion

In this paper, we have proposed recursive algorithms for the optimal power allocation of energy harvesting device equipped with multiple antennas and extend to a hybrid system with both harvested energy and grid power.

By applying the proposed GWF repeatedly and comparing the water level of the processing window with the previous epoch, RGWFM outputs the solution of the power allocation epoch by epoch. RGWFMn and RGWFMt are originally constructed based on RGWFM with additional comparison: whether the required information transmission of B bits being achieved to efficiently compute the accurate solutions to the minimum transmission completion time problems. Therefore, although these optimization problems are nonconvex and nonsmooth, they can also be solved by the proposed algorithms. Further, the computation of all these proposed algorithms is finite. We have also obtained and strictly proven optimality of the proposed algorithms. Numerical examples are provided to illustrate the steps to obtain the optimal solutions by these proposed algorithms. Significant gains of the achieved throughput can be observed by using the proposed algorithms over the typical optimization method.

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