Recursive Waterfilling for Wireless Links With Energy Harvesting Transmitters

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***Abstract*—Energy harvesting is often used for green communications. The problem of power allocation is then to maximize the throughput, taking into account the fact that channel conditions and energy sources are time varying. In particular, for the constraints of the target problem, besides the allocated power values being nonnegative, the successively harvested energy sum leads to the triangle coefficient matrix of the power sum constraints. In this paper, we propose a geometric waterfilling (GWF) algorithm in place of the conventional waterfilling (CWF) algorithm for power allocation with a sum power constraint. We then recursively apply the GWF as a functional block to sequentially solve the power allocation problem for energy harvesting transmission in a fading channel. This algorithm is referred to as RGWF. The proposed RGWF is further extended to solving the minimization of the transmission completion time (referred to as RGWFn) by inserting a condition to check if the preset information transmission data bits are achieved. Since RGWF is defined by recursion and along natural progress of time, we can compute a family of solutions for subprocesses from epoch 1 to epoch k, for $k = 1, \ldots, K$, where K is the index of the final epoch for the entire process. Thus, RGWF can be utilized for efficiently carrying out the computation of RGWFn. RGWF and RGWFn belong to dynamical recursive algorithms. Compared with the existing results in the open literature, the proposed algorithms have distinguished features: 1) They provide the exact optimal solutions via efficient finite computation under the recursive category, and 2) the optimality of the proposed algorithms is strictly proven. Numerical examples are provided to illustrate the procedures to obtain the optimal power allocation by using the proposed algorithms.**

***Index Terms*—Exact optimal solution, geometric approach, green communications, harvesting energy, optimization methods, radio resource management, recursion, waterfilling.**

I. Introduction

THERE has been recent research effort on understanding data transmission with an energy harvesting transmitter that has a rechargeable battery for green communications [1]–[4]. The system is assumed to consist of a sequence of epochs. For each epoch, an event occurs, which may be the consequence of channel fading gain variation or new energy being harvested, or both. This setting leads to new design insights into a wireless link with a rechargeable transmitter and fading channels.

Generally, the incoming energy can be stored in the battery of the rechargeable transmitter for future use. However, it cannot be used for previous epochs. This point is called causality. Often, the battery owns great storage capacity, and it is hardly filled fully from the incoming harvested energy. This assumption is taken into account in this paper, i.e., the maximum energy capacity of the battery $E_{\max} \gg 0$. This assumption lays down a foundation to solve the cases of finite $E_{\max}$. In this setting, we can compute optimal dynamic transmission schemes that adapt the instantaneous transmit power to the variations in the energy and fade levels.

In recent years, energy harvesting green communication has attracted substantial research attention. In [1], data transmission with energy harvesting sensors is considered, and the recursive policy for controlling admissions into the data buffer is derived using a dynamic programming framework. Dynamic programming can offer an optimal closed-loop control, but it may meet with the curse of dimensionality [5]. In [2], energy management policies that stabilize the data queue are proposed for single-user communication and under a linear approximation. Some delay optimality properties are derived. In [3] and references therein, optimization approaches are considered to attempt to obtain the maximum throughput over additive white Gaussian noise and fading channels. Successively, in [4], throughput optimal energy allocation is studied for energy harvesting systems in a time-constrained slotted setting. The same objective function is investigated in [3] and [4] using offline machinery and waterfilling approach, which comes directly from the Karush–Kuhn–Tucker (KKT) conditions [6] of the target problem. In [7], the optimality of a variant of the back pressure algorithm using energy queues is discussed. In [8], energy harvesting transmitters with batteries of finite energy are considered, and the problem of throughput maximization is solved offline in a (nonfading) static channel. Recently, Gong *et al.* [9] have investigated the issue of optimal power allocation for energy harvesting and power grid coexisting wireless communication systems and proposed the related method.

In this paper, we propose optimal recursive waterfilling algorithms to solve the problems with energy harvesting constraints. With waterfilling, more power is allocated to the channels with higher gains to maximize the sum data rate of all the channels [10]. The conventional way to solve the waterfilling problem is to solve the KKT conditions and then find the water level(s) and the solutions. In this paper, we propose an approach from a simple geometric view of waterfilling, construct a solution, and

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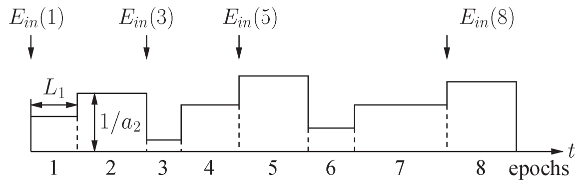

Fig. 1. System model. $K = 8$ epochs in $(0, T]$.

then prove its optimality. We refer to this approach as geometric waterfilling (GWF). Due to the complexity of solving the KKT conditions of the problem in conventional waterfilling (CWF), the proposed GWF is easier to compute than the CWF and reveals more useful information.

The proposed GWF is used as a functional block to solve the target problem with energy harvest in fading channels. Successive energy harvesting and fading gain variations lead to more complicated constraints for the optimization problem. In addition to the nonnegative allocated power, the successively incremental harvested energy sum leads to a triangle coefficient matrix of the constraints. Due to this triangle coefficient matrix, the proposed algorithm can utilize recursion machinery, with GWF being used as a functional block. Thus, it is referred to as RGWF. Compared with the existing results, the proposed RGWF owns two characteristics: 1) It efficiently provides the exact optimal solution recursively with finite computation, and 2) its optimality is strictly proven. Due to the usage of recursion, the solutions to a family of the maximum throughput problems for subprocesses from epoch 1 to epoch k, for $k = 1, \ldots, K$, can be efficiently computed, where K is the index of the final epoch in the process. In addition, since the circular definition of an algorithm has been avoided, our approach is more strict. Furthermore, RGWF is extended to solving the minimum transmission time problem, with the objective function of minimizing the transmission time for transmitting the given amount of information, measured as B bits. By including an "**if**" clause in the algorithm to check if B bits are transmitted, the algorithm RGWFn is formed from RGWF to solve the problem. We also prove the optimality of RGWFn to solve the transmission time minimization problem without using the KKT condition for this nonconvex optimization problem in the mixed continuous and discrete variables.

In the remainder of this paper, the system model and problem statement are presented in Section II. The proposed GWF with a sum power constraint is discussed in Section III. The proposed RGWF and RGWFn algorithms and their optimality are investigated in Sections IV and V, respectively. Numerical examples are presented in Section VI. Section VII concludes this paper.

II. System Model and Problem Statement

Here, we introduce the system model and the transmission throughput maximization problem with energy harvesting in a fading channel. For convenience and without loss of generality, the process is assumed to be a discrete time process.

As shown in Fig. 1, we consider the time period from $(0, T]$, including K epochs. Each epoch is the result of either the channel state change or the harvest energy arrival or the occurrence of both events. We use L_i and a_i to denote the time duration and the fading gain of the ith epoch. At the beginning of the ith epoch, the energy arrival is denoted by $E_{in}(i)$, and the event of energy arrival is depicted as $E_{in}(i) > 0$.

The objective is to maximize the number of bits transmitted by the deadline T, i.e., within the K epochs. The optimal power management strategy is such that the transmit power is constant in each epoch. Therefore, let us denote the transmit power in epoch i by s_i, for $i = 1, \ldots, K$.

We have causality constraints due to energy arrivals and an $E_{\max}$ constraint due to finite battery size. Hence, the optimization problem in this fading case becomes [3]

$$\begin{aligned} \max_{\{s_i\}_{i=1}^K} \quad & \sum_{i=1}^{K} \frac{L_i}{2} \log(1 + a_i s_i) \\ \text{subject to} \quad & 0 \leq s_i \ \forall\, i \\ & \sum_{i=1}^{l} L_i s_i \leq \sum_{i=1}^{l} E_{in}(i), \text{ for } l = 1, \ldots, K \\ & \sum_{i=1}^{l} E_{in}(i) - \sum_{i=1}^{l} L_i s_i \leq E_{\max} \\ & \text{for } l = 1, \ldots, K-1 \end{aligned} \tag{1}$$

where if we interpret the observed properties of the optimal power allocation scheme as a waterfilling scheme, s_i units of water is filled into a rectangle container with a bottom width as $L_i/2 \,\forall\, i$. With the assumption of $E_{\max} \gg 0$, the last constraint in (1) is relaxed. Note that the last power sum constraint in this relaxed problem [$l = K$ in (1)] is of equality. Furthermore, for unifying parameter notation, through a change in variables, we can obtain an equivalent problem to (1) under the relaxation condition of $E_{\max} \gg 0$ as follows:

$$\max_{\{s_i\}_{i=1}^K} \quad \sum_{i=1}^{K} w_i \log(1 + a_i s_i) \tag{2}$$

$$\text{subject to} \quad 0 \leq s_i \ \forall\, i \tag{3}$$

$$\sum_{i=1}^{l} s_i \leq \sum_{i=1}^{l} E_{in}(i) \ \forall\, l \tag{4}$$

where $w_i <= L_i/2$, $a_i <= a_i/L_i$, and $s_i <= L_i s_i$. Note that the symbol $<=$ is the assignment operator.

To find the solution to problem (2), the CWF approach usually starts from the KKT conditions of the problem as a group of the optimality conditions, and then, the following system in both the variables $\{s_i\}$ and the dual variables can be written as

$$\begin{cases} s_i = \left(\frac{w_i}{\sum_{j=i}^{K} \lambda_j} - \frac{1}{a_i} \right)^+, \text{ for } i = 1, \ldots, K \\ \sum_{i=1}^{l} w_i \left(\frac{1}{\sum_{j=i}^{K} \lambda_j} - \frac{1}{a_i w_i} \right)^+ \leq \sum_{i=1}^{l} E_{in}(i) \ \forall\, l \\ \lambda_j \geq 0 \ \forall\, j \end{cases} \tag{5}$$

where λ_j is the dual variable corresponding to the jth sum power constraint for any j. The solution to system (5) is the solution to problem (2). However, it is not easy to solve (5). That is to say, only observing or using monotonicity of $(1/(\sum_{j=i}^{K} \lambda_j))$ with respect to i, in the first KKT condition that is related with s_i, is not enough to compute the solution. The set of $\{\lambda_j\}$ or $\{\sum_{j=i}^{K} \lambda_j\}$ needs to satisfy other KKT conditions. Unlike the conventional approach, our proposed algorithm effectively solves the target problem and also solves the KKT conditions as a by-product based on the proposed geometric approach.

III. Geometric Waterfilling With Sum Power Constraint

In [11], we presented a GWF approach and its application to solve generalized radio resource allocation problems. The GWF approach eliminates the procedure to solve the nonlinear system for the water level and provides explicit solutions and helpful insights into both the problem and the solution. In this paper, we will use GWF as a functional block to solve target problem (2). For self-explanatory, the basic concept of GWF is reviewed in this section.

A. Problem, Algorithm, and Optimality

Let L and K be two positive integers; $L \leq K$ denote the index of the starting channel and the ending channel, respectively; and $K - L + 1$ denote the total number of channels. Often, L is assigned to be 1. The waterfilling problem can be abstracted and generalized into the following problem: Given $P > 0$, as the total power or volume of the water, the allocated power and the propagation path gain for the ith channel are given as s_i and a_i, respectively, $i = L \ldots K$. Furthermore, given the weighted coefficient $w_i > 0\ \forall i$, associated with $\{a_i w_i\}_{i=L}^{K}$ assumed to be in decreasing order (the indexes can be arbitrarily renumbered to satisfy this condition), we find that

$$\max_{\{s_i\}_{i=L}^{K}} \quad \sum_{i=L}^{K} w_i \log(1 + a_i s_i) \tag{6}$$

$$\text{subject to} \quad 0 \leq s_i,\ \forall i \tag{7}$$

$$\sum_{i=L}^{K} s_i = P. \tag{8}$$

Since the constraints are that 1) the allocated power has to be nonnegative and 2) the sum of the power equals P, problem (6) is called the waterfilling (problem) with a sum power constraint.

Fig. 2(a)–(d) shows an illustration of the proposed GWF algorithm. Suppose there are four steps/stairs ($L = 1$, $K = 4$) inside a water tank. For the conventional approach, the dashed horizontal line, which is the water level μ, needs to be determined first, and then, the power allocated (water volume) above the step is solved.

Let us use w_i to denote the width of the ith step. For the ith step, the allocated power s_i represents the area from the step to the surface of the water (if this step is under water). The term $1/a_i$ represents the area from the step to the bottom of the tank. Therefore, the "step depth" of the ith step d_i (the height of the ith step to the bottom of the tank) can be expressed as

$$d_i = \frac{1}{a_i w_i}, \text{ for } i = L, L+1, \ldots, K \tag{9}$$

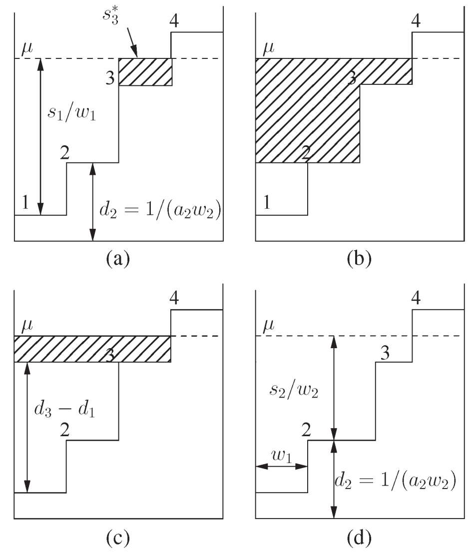

Fig. 2. Proposed GWF algorithm. (a) Water level step $k^* = 3$, allocated power for the third step s_3^*, and step/stair depth $d_i = (1/(a_i w_i))$. (b) $P_2(k)$ (shadowed area, representing the total water/power above step k) when $k = 2$. (c) $P_2(k)$ when $k = 3$. (d) Weights as the widths.

where $\{a_i, w_i\}$ in (9) is the same as those in (6). Due to the monotonicity of the sequence $\{a_i w_i\}$, the step depth of the stairs indexed as $\{L, \ldots, K\}$ is monotonically increasing.

Instead of trying to determine the water level μ, which is a nonnegative real number, we aim to determine the water level step, which is an integer number from L to K, which is denoted by k^*, as the highest step under water. Based on the result of k^*, we can write out the solutions for power allocation instantly.

Fig. 2(a) shows the concept of k^*. Since the third level is the highest level under water, we have $k^* = 3$. The shadowed area denotes the allocated power for the third step by s_3^*.

In the following, $P_2(k)$, which is the water volume above step k, can be obtained considering the step depth difference and the width of the stairs as

$$P_2(k) = \left[P - \sum_{i=L}^{k-1} (d_k - d_i) w_i\right]^+, \text{ for } k = L, \ldots, K \tag{10}$$

where $(x)^+ = \max\{0, x\}$.

As an example in Fig. 2(c), the water volume above step 1 and below step 3 with width w_1 can be found as the step depth difference $(d_3 - d_1)$ multiplying the width of the step w_1. Therefore, the corresponding $P_2(k = 3)$ can be expressed as

$$P_2(k = 3) = [P - (d_3 - d_1) w_1 - (d_3 - d_2) w_2]^+$$

which is an expansion of (10). Then, we have the following proposition.

Proposition 1: The explicit solution to (6) is

$$\begin{cases} s_i = \left[\frac{s_{k^*}}{w_{k^*}} + (d_{k^*} - d_i)\right] w_i, & L \leq i \leq k^* \\ s_i = 0, & k^* < i \leq K \end{cases} \tag{11}$$

where

$$k^* = \max\left\{k \middle| P_2(k) > 0, \quad L \le k \le K\right\}, \tag{12}$$

the power level for this step is

$$s_{k^*} = \frac{w_{k^*}}{\sum_{i=L}^{k^*} w_i} P_2(k^*) \tag{13}$$

and the corresponding water level can be expressed as

$$\mu = \frac{\frac{1}{a_{k^*}} + s_{k^*}}{w_{k^*}} = \frac{\frac{1}{a_i} + s_i}{w_i}, \text{ as } L \le i \le k^*. \tag{14}$$

The proof of Proposition 1 can be found in [11].

In summary, for GWF, the first step is to calculate $P_2(k)$, then find the water level step k^* from (12), which is the maximal index making $P_2(k)$ positive. The corresponding power level for this step, i.e., s_{k^*}, can be obtained by applying (13). Then, for those steps with an index higher than k^*, the powers are assigned with zeros. For those steps below k^*, the powers are assigned as in (11). The first term (s_{k^*}/w_{k^*}) inside the square bracket denotes the depth of the k^*th step to the surface of the water. The second term inside the square bracket denotes the step depth difference of the k^*th step and the ith step. Therefore, the sum inside the square bracket means the depth of the ith step to the surface of the water. When this quantity is multiplied with the width of this step, the area of the water above this step (allocated power) can be then readily obtained.

From Proposition 1, when k^* is obtained, $P_2(k^*)$ is given. Then, it is memorized and only multiplied by a constant to compute s_{k^*}. Thus, how to search k^* is a key point for the proposed GWF, and the procedure of GWF is stated as follows.

1) Initialize $W_s = 0$; $P_M = P^* = P$; $i = L$.
2) Compute $W_s <= W_s + w_i$; $P^* <= P^* - (d_{i+1} - d_i)W_s$. Then, $i <= i + 1$. Note that it has been claimed that the symbol "<=" represents the assignment operation.
3) If $P^* > 0$ and $i \le K$, $P_M = P^*$, and repeat step 2); else, output $k^* = i - 1$ and $s_{k^*} = (w_{k^*}/W_s)P_M$.

Thus, solution $\{s_i\}$ is obtained. We can observe that $(s_{k^*}/w_{k^*}) + d_{k^*}$ is the water level due to $(s_{k^*}/w_{k^*}) + d_{k^*} = (s_i/w_i) + d_i$ for $L \le i \le k^*$. In addition, GWF can be regarded as a mapping from the point of parameters $\{L, K, \{w_i\}_{i=L}^K, \{a_i\}_{i=L}^K, P\}$ to solution $\{s_i\}_{i=L}^K$ and the important water level step index k^* [11]. That is to say, it can be written as a formal expression as follows:

$$\left\{\{s_i\}_{i=L}^K, k^*\right\} = GWF\left(L, K, \{w_i\}_{i=L}^K, \{a_i\}_{i=L}^K, P\right). \tag{15}$$

Since we often use the first part, i.e., $\{s_i\}_{i=L}^K$ from GWF previously mentioned, we also write

$$\{s_i\}_{i=L}^K = GWF\left(L, K, \{w_i\}_{i=L}^K, \{a_i\}_{i=L}^K, P\right)|_I. \tag{16}$$

Note that, for concision and without confusion from the context, we may write the right-hand side of the expression as GWF(L, K) to emphasize time stages from L to K.

B. Complexity Analysis

As stated in [12] (see Section III), the conventional WF algorithm has an exponential worst case complexity of 2^K loops, where K is the number of channels, although the channel gains have been sorted in decreasing order. Pointing to this case, Palomar in [12] proposed an improved algorithm with the worst-case complexity of K loops. Since each loop consists of multiple arithmetic and logical operations, here, we use the total number of operations as a measure of complexity (see [13, Ch. 8]).

The CWF approach has a worst case complexity of K loops, i.e., total $O(K^2)$ fundamental arithmetic and logical operations under the $2(K+1)$ memory requirement and the sorted parameters $\{w_k a_k\}_{k=1}^K$ (e.g., see [14, p. 137] for more details).

The proposed GWF algorithm occupies less computational resource. It is seen that it needs K loops at most to search k^* and four arithmetic operations and two logical operations to complete each loop. Thus, the worst case computational complexity of the proposed solution is $8K + 3$ (from the operations of $6K + 3 + 2K$) fundamental arithmetical and logical operations under the $2(K+1)$ memory units to store $\{d_i\}$, $\{w_i\}$, W_s, and P_M.

IV. Proposed Recursive Geometric Waterfilling

Here, we propose a novel approach to solve problem (2) using our proposed GWF approach recursively (RGWF). The constraint in (4) can be expanded into a matrix form as

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & 0\cdots & 0 \\ \cdot & \cdot & \cdot & \\ 1 & 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_K \end{pmatrix} \leqq \begin{pmatrix} \sum_{i=1}^1 E_{in}(i) \\ \sum_{i=1}^2 E_{in}(i) \\ \vdots \\ \sum_{i=1}^K E_{in}(i) \end{pmatrix}.$$

We noticed that the coefficient matrix forms a triangle matrix.

A. Algorithm RGWF

The proposed RGWF(K) is stated as in the following algorithm description.

Note that in the first line of Algorithm 1, K represents the total number of epochs, and L denotes the index of the current processing epoch. Beginning from line 3, RGWF sequentially processes from the second epoch to the Kth epoch. The inner "**for**" loop (lines 6–17) updates power levels for the current processing epoch (L) and its previous $L-n$ epochs to form a processing window. The GWF algorithm is applied to this window to find a common water level. A summation is used in line 8. The function of this step is to sum up all the harvested energy/power in the processing window. If the lower limit of the summation is greater than the upper limit, the result of this summation is defined as zero. Then, the "**if**" clause compares the water level of this processing window with the previous epoch's water level. If the water level nondecreasing condition is satisfied, then output RGWF(L), and move to processing the next epoch. If the water level nondecreasing condition is not satisfied, the window is expanded by one epoch on the left side. Through this mechanism, solution $\{s_i^*\}_{i=1}^K$ is obtained as RGWF(K) within finite loops.

Algorithm 1 Pseudocode for RGWF

1: Initialize:
    $L = 1, K, P = E_{in}(1), w_1, a_1$;
2: Output the result for epoch 1:
    $\text{RGWF(L)} = \text{GWF}(1,1)|_I = s_1^* = E_{in}(1)$;
3: **for** $L = 2{:}1 : K$ **do**
4:     Input: $\{E_{in}(L), \{w_L, a_L\}\}$;
5:     $\{s'_k\}_{k=1}^{L-1} = \text{RGWF}(L-1)$;
6:     **for** $n = L : -1{:}1$ **do**
7:         $W = \{w_j\}_{j=n}^L$; $A = \{a_j\}_{j=n}^L$;
8:         $S_T = \sum_{j=n}^{L-1} s'_j + E_{in}(L)$;
9:         $\{\{s_{k^*}^*\}_{k=n}^L, k^*\} = \text{GWF}(n, L, W, A, S_T)$;
10:         **if** $n > 1$ **then**
11:             $k_\text{e}^* = \max\{k | s'_k > 0, 1 \le k \le n-1\}$,
             else $k_\text{e}^* = 1$;
12:         **end if**
13:         **if** $(1/a_{k^*} w_{k^*}) + (s_{k^*}^*/w_k^*) \ge (1/a_{k_\text{e}} w_{k_\text{e}}) + (s'_{k_\text{e}}/w_{k_\text{e}})$ **then**
14:             output: $\text{RGWF(L)}|_I = \{s'_1, \ldots, s'_{n-1}, s_n^*, \ldots, s_L^*\}$,
15:             Move to next epoch, i.e., go to line 16;
16:         **end if**
17:     **end for**
18: **end for**
19: Output RGWF(K): $\{s_k^*\}_{k=1}^K = \text{RGWF(K)}$.

Fig. 3 is an illustration of the inner "**for**" loop of RGWF. It is assumed that the current processing epoch is $L = 6$ with the assumption of $E_{in}(6) = 0$. The optimal power allocation for the first five epochs has been completed, as shown in the shadowed areas in Fig. 3(a). Epoch 6 is now under processing.

Based on line 9, since there is no energy harvested in epoch 6, the power level for epoch 6 is zero, and the water level is just the fading level. Lines 10–12 calculate that $k_e^* = 5$, and then, line 13 compares the water level of the current processing window with that of the k_e^*th epoch. Since the comparison in line 13 does not hold, the algorithm goes back to line 6 by decreasing n to 5, and then, the processing window is extended to including epochs 5 and 6, as shown in Fig. 3(b). Fig. 3(b) also shows the power allocation from GWF(5, 6) in line 9 as the horizontal-wave shadowed areas. Still, the comparison of the water level nondecreasing condition for the window [5, 6] in line 13 does not hold, and the algorithm returns to line 6 again by decreasing $n = 4$. As shown in Fig. 3(c), the processing window is epochs 4–6. The water level nondecreasing condition still is not satisfied. The processing window is extended from epochs 3–6, as shown in Fig. 3(d). With the new water level in the processing window, the water level nondecreasing condition up to epoch 6 is satisfied. As a result, RGWF($L = 6$) is solved, which is recursively obtained from RGWF($L - 1 = 5$).

Through this mechanism, solution $\{s_i^*\}_{i=1}^K = \text{RGWF(K)}$ is recursively obtained.

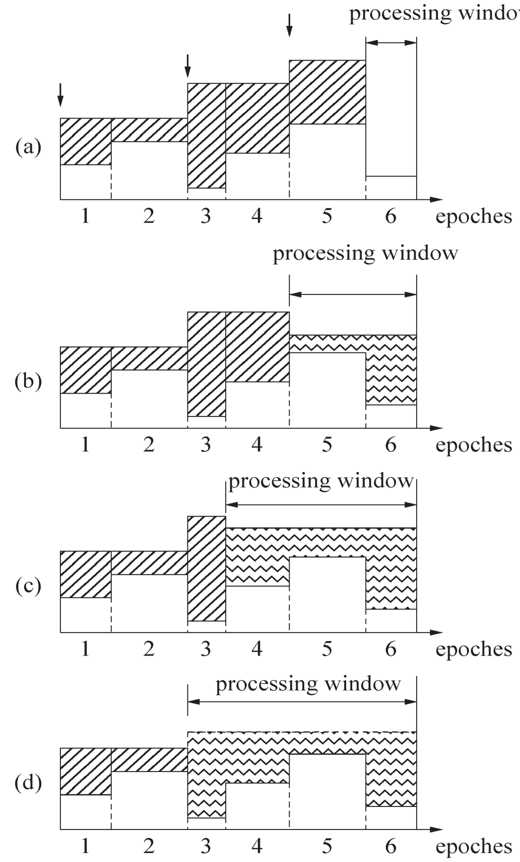

Fig. 3. Inner loop of RGWF (lines 6–18 for $L = 6$), harvested energy having been allocated up to epoch 5; horizontal-wave shadowed areas denote power allocation for the processing window. (a) $n = 6$, $k_e^* = 5$. (b) $n = 5$, $k_e^* = 4$. (c) $n = 4$, $k_e^* = 3$. (d) $n = 3$, $k_e^* = 2$.

B. Optimality of RGWF

The proposed algorithm eliminates the procedure to solve nonlinear system (5) in multiple variables and dual variables, provides exact solutions via finite computation steps, and offers helpful insights into the problem and the solutions. To guarantee optimality of RGWF, we have the following proposition.

Proposition 2: RGWF can compute the optimal exact solution to problem (2) within finite loops.

Proof of Proposition 2: From the algorithm RGWF(K), there exists n_1, where $1 \le n_1 \le K - 1$, and $\{s_i^*\}_{i=1}^{n_1} = [\text{RGWF}(K)]|_{\{1,\ldots,n_1\}}$. Thus, there are the nonnegative Lagrange dual variables $\{\lambda_i\}_{i=1}^{n_1}$ and $\{\mu_i\}_{i=1}^{n_1}$ such that KKT conditions of the restriction of the optimization problem RGWF(K) to the set $\{1, \ldots, n_1\}$ hold. This restriction indicates a subproblem, i.e.,

$$\max_{\{s_i\}_{i=1}^{n_1}} \quad \sum_{i=1}^{n_1} w_i \log(1 + a_i s_i) \tag{17}$$

$$\text{subject to} \quad \sum_{i=1}^{l} s_i \le \sum_{i=1}^{l} E_{in}(i) \tag{18}$$

$$0 \le s_i, \text{ as } 1 \le l \le n_1 - 1 \tag{19}$$

$$\sum_{i=1}^{n_1} s_i = \sum_{i=1}^{n_1} s_i^*. \tag{20}$$

Further, $\{\lambda_i, \mu_i\}$ correspond to the ith sum power constraint and the power nonnegativeness constraint, respectively. On the other hand, $\{\{s_i^*\}_{i=n_1+1}^K, k^*\} = \text{GWF}(n_1+1, K, \{w_j\}, \{a_j\}, \sum_{j=n_1+1}^K s_j^*)$. Thus, there are also the nonnegative Lagrange dual variables λ and $\{\mu_i\}_{i=n_1+1}^K$ that are the KKT conditions of the following subproblem:

$$\max_{\{s_i\}_{i=n_1+1}^K} \quad \sum_{i=n_1+1}^{K} w_i \log(1 + a_i s_i) \tag{21}$$

$$\text{subject to} \quad 0 \le s_i \; \forall i \tag{22}$$

$$\sum_{i=n_1+1}^{K} s_i = \sum_{i=n_1+1}^{K} s_i^*. \tag{23}$$

Since $\text{GWF}(n_1+1, K)$ has one sum power constraint and specific finite-loop operations, we can assign

$$\lambda_K = \lambda = \frac{1}{\frac{1}{a_{k^*} + w_{k^*}} + \frac{s_{k^*}}{w_{k^*}}} \tag{24}$$

$$\lambda_{K-1} = \cdots = \lambda_{n_1+1} = 0 \tag{25}$$

with the fine k^*, as the minimum positive step index of set $\{n_1+1, \ldots, K\}$, where the adjective "fine" expresses that k^* can be used to clarify whether the allocated power to be positive or zero and determine the water level at once. Moreover, due to the characteristics of the loop transition from n_1+1 to n_1 during carrying out $\text{RGWF}(n_1)$ and the points previously mentioned, it is seen that the feasible solution of $\{s_i^*\}_{i=1}^K$, which is computed by RGWF(K), is indeed the optimal solution to (2)–(4).

Therefore, Proposition 2 is proved.

Remark 1: RGWF is an optimal dynamic power distribution process. The dynamics of this process is shown by the generalized varying structure state equation

$$\text{RGWF}(L+1) = [[\text{RGWF}(L)]|_{\Lambda_1}, [\text{GWF}(n, L+1)|_I]\,|_{\Lambda_2}] \qquad \text{for } L = 1, \ldots, K-1$$

where n is the index of the starting epoch of the currently processing window (i.e., it satisfies line 13 of RGWF), the set Λ_1 denotes $\{s'_k\}_{k=1}^{n-1}$ and is referred to in the description or definition of RGWF, and the set Λ_2 denotes $\{s_k\}_{k=n}^{L+1}$ and is referred to in (16). In this process, $\text{RGWF}(L)$ can be regarded as the generalized system state at the time stage (or epoch) L; $\text{GWF}(n, L+1)$ can be regarded as the generalized system control at the time stage (or epoch) L; and then $\text{RGWF}(L+1)$, as a state at the next time stage, can be derived or determined from the previous state and control. Due to the optimality of $\text{RGWF}(L)$ from Proposition 2, for any L, therefore, the proposed algorithm is indeed an optimal dynamically recursive waterfilling algorithm with high efficiency.

V. Transmission Completion Time Minimization

In previous section, RGWF was discussed to efficiently solve the throughput maximization problem. Here, RGWF is extended to solve the transmission completion time minimization problem.

Now, assume that the transmitter has B bits to be transmitted to the receiver. Our objective now is to minimize the time required to transmit these B bits. This problem is called the transmission completion time minimization problem. In [8] and [15], this problem is formulated and solved for an energy harvesting system in a nonfading environment. In [3], the problem is attempted to be solved offline in a fading channel. In this paper, we use the proposed RGWF to solve the problem in a fading channel with recursive computation, which is referred to as RGWFn.

The transmission completing time minimization can be stated as follows, assuming N to be a positive integer and $N \le K$:

$$\min_{\{\{s_i\}_{i=1}^N, N\}} \quad N \tag{26}$$

$$\text{subject to} \quad \sum_{i=1}^{N} w_i \log(1 + a_i s_i) = B \tag{27}$$

$$0 \le s_i \; \forall i \tag{28}$$

$$\sum_{i=1}^{l} s_i \le \sum_{i=1}^{l} E_{in}(i), \; l = 1, \ldots, N. \tag{29}$$

The proposed RGWFn is presented in the algorithm description shown below.

Compared with the steps of RGWF, in the first line of the algorithm, RGWFn introduces B as a parameter, the others being the same. In line 8, RGWFn sequentially processes from the second epoch to the Kth epoch to output the optimal value N^* and its optimal solution, i.e., $\{\text{RGWFn}(N^*), N^*\}$. Similarly, the inner "**for**" loop updates power levels for the current processing epoch (L) and its previous $L-n$ epochs to form a processing window. The GWF algorithm is also applied to this window to find a common water level. Note that a new "**if**" clause is inserted into the outer level "**if**" clause (for the water level nondecreasing condition check). The function of this inner "**if**" clause is to check whether the transmitted bits reach B. Therefore, it is the normal exit of the algorithm (lines 23–28). For convenience, the condition of this new "**if**" clause is called the criterion of RGWFn. This is also due to the importance of the criterion in the following proposition.

Algorithm 2 RGWFn, Based on RGWF

1: Initialize: $L = 1$, K, B, $E_{in}(1)$, w_1, a_1;
2: Output the result for epoch 1:
    $s'_1 = \text{RGWF}(1) = E_{in}(1)$,
3: **if** $\sum_{i=1}^{1} w_i \log(1 + a_i s'_i) \ge B$ **then**
4:     $\text{RGWFn}(1) = s_1^* = (1/a_L)(2^{(B/w_L)} - 1)$;
5:     $N^* = 1$;
6:     Exit the algorithm;
7: **end if**
8: **for** $L = 2{:}1 : K$ **do**
9:     Input: $\{E_{in}(L), w_L, a_L\}$;
10:     $\{s'_k\}_{k=1}^{L-1} = \text{RGWF}(L-1)$;

11: **for** $n = L : -1 : 1$ **do**
12: $W = \{w_j\}_{j=n}^{L}$; $A = \{a_j\}_{j=n}^{L}$;
13: $S_T = \sum_{j=n}^{L-1} s'_j + E_{in}(L)$;
14: $\{\{s_k^*\}_{k=n}^{L}, k^*\} = \text{GWF}(n, L, W, A, S_T)$;
15: **if** $n > 1$ **then**
16: $k_{\text{e}}^* = \max\{k | s'_k > 0, 1 \le k \le n-1\}$, else $k_{\text{e}}^* = 1$;
17: **end if**
18: **if** $(1/a_{k^*} w_{k^*}) + (s_{k^*}/w_k^*) \ge (1/a_{k_{\text{e}}} w_{k_{\text{e}}}) + (s'_{k_{\text{e}}}/w_{k_{\text{e}}})$ **then**
19: $\text{RGWF}(L) = \{s'_1, \ldots, s'_{n-1}, s_n^*, \ldots, s_L^*\}$;
20: $T_1 = \sum_{i=1}^{n-1} w_i \log(1 + a_i s'_i)$;
21: $T_2 = \sum_{i=n}^{L} w_i \log(1 + a_i s_i^*)$;
22: $T_3 = \sum_{i=n}^{L-1} w_i \log(1 + a_i s_i^*)$;
23: **if** $T_1 + T_2 \ge B$ **then**
24: $B_1 = B - T_1 - T_3$;
25: $s_L^* = (1/a_L)(2^{(B_1/w_L)} - 1)$;
26: $\text{RGWFn}(L) = \{s'_i\}_{i=1}^{n-1} \cup \{s_j^*\}_{j=n}^{L}$;
27: $N^* = L$;
28: Exit the algorithm;
29: **end if**
30: Move to next epoch, i.e., go to line 33;
31: **end if**
32: **end for**
33: **end for**

Proposition 3: If there does not exist L such that the criterion in RGWFn

$$\sum_{i=1}^{n-1} w_i \log\left(1 + a_i s'_i\right) + \sum_{i=n}^{L} w_i \log\left(1 + a_i s_i^*\right) \ge B \qquad (30)$$

holds, where the symbols in (30) keep the same meaning as those in the statement of RGWFn, then there is no solution to problem (26). If the criterion holds, then the obtained N^* is the optimal value, and $\{\text{RGWFn}(N^*), N^*\}$ is the exact optimal solution.

Proof of Proposition 3: For the given B, if there does not exist L such that the criterion in RGWFn

$$\sum_{i=1}^{n-1} w_i \log\left(1 + a_i s'_i\right) + \sum_{i=n}^{L} w_i \log\left(1 + a_i s_i^*\right) \ge B$$

holds, it implies that the optimal value of problem (2) is strictly less than B, corresponding to Proposition 2. Thus, the first constraint of problem (26) never holds. Then, there is no solution to problem (26).

Now, assume that there exist N^* and $\text{RGWF}(N^*)$ such that

$$\sum_{i=1}^{n-1} w_i \log\left(1 + a_i s'_i\right) + \sum_{i=n}^{N^*} w_i \log\left(1 + a_i s_i^*\right) \ge B$$

where

$$\text{RGWF}(N^*) = \left\{s'_1, \ldots, s'_{n-1}, s_n^*, \ldots, s_{N^*}^*\right\}.$$

According to the obtained N^* from the RGWFn algorithm, the optimal value of the problem

$$\begin{aligned} \max_{\{s_i\}_{i=1}^{N}} \quad & \sum_{i=1}^{N} w_i \log(1 + a_i s_i) \\ \text{subject to} \quad & 0 \le s_i \; \forall\, i \\ & \sum_{i=1}^{l} s_i \le \sum_{i=1}^{l} E_{in}(i) \text{ for } l = 1, \ldots, N \end{aligned}$$

is less than B, where $\text{RGWF}(N)$ is the optimal solution to this problem, for $N = 1, \ldots, N^* - 1$. Hence, the optimal value of problem (26) is not less than N^*. Stemming from the statement of RGWFn, $\{\text{RGWFn}(N^*), N^*\}$ is a feasible solution to problem (26), and further, N^* is the evaluated objective value of problem (26) at $\{\text{RGWFn}(N^*), N^*\}$. Thus, N^* is a feasible value. Together with the previously mentioned fact that the optimal value of problem (26) is not less than N^*, as a result, N^* is the optimal value, and $\{\text{RGWFn}(N^*), N^*\}$ is the exact optimal solution to problem (26).

Therefore, Proposition 3 is proved.

Remark 2: RGWFn is an optimal dynamic progressive process to compute the transmission completion time minimization. This progressive process is ended at the current epoch and then outputs the minimum completing time once the criterion is satisfied. Hence, it does not need the information/solution of the entire process of the problem(s).

In addition, given the lengths of epochs, i.e., $\{L_i\}$, due to $w_i = (L_i/2) \; \forall\, i$, the minimum transmission completion time duration is $2\sum_{k=1}^{N^*} w_i$, which can be computed by finding N^*.

VI. Numerical Examples

Here, we provide numerical examples for illustrating the procedures of the proposed algorithms (RGWF and RGWFn). Since RGWF uses the recursion mechanism, it can effectively compute the exact optimal solution through finite computation for every subprocess that starts from epoch 1 and ends at epoch i, as $i = 1, \ldots, K$. This point can be utilized for RGWFn to efficiently compute the solution to the minimum transmission completion time problem.

The proposed algorithm scans the epochs sequentially to obtain the optimal power allocation. The achieved sum data rate is not plotted, since we have shown that the proposed algorithms provide the optimal solution. For simple illustration, we assume that there are three epochs, each with the time length ($L_i = 2$, $i = 1, 2, 3$), i.e., the same unit weight ($w_i = 1, i = 1, 2, 3$) in (2).

Example 1: Suppose the fading profiles for the three epochs are $a_1 = 1$, $a_2 = (1/2)$, and $a_3 = (1/3)$. At the beginning of each epoch, unit energy is harvested ($E_{in}(i) = 1$, $i = 1, 2, 3$). Then, the maximizing throughput problem is

$$\max_{\{s_i\}_{i=1}^{3}} \quad \sum_{i=1}^{3} \log(1 + a_i s_i) \qquad (31)$$

$$\text{subject to} \quad 0 \le s_l \qquad (32)$$

$$\sum_{i=1}^{l} s_i \le l, \; l = 1, 2, 3. \qquad (33)$$

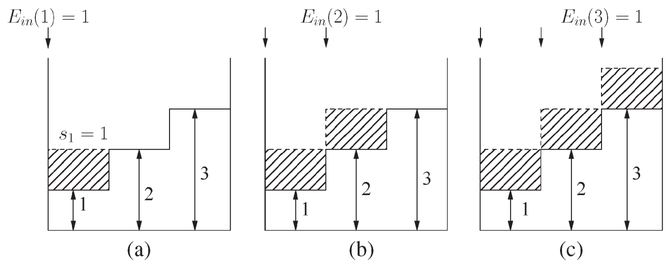

Fig. 4. Procedures to solve Example 1. (a) $s_1 = 1$. (b) $s_1 = 1$, $s_2 = 1$. (c) $s_1 = 1, s_2 = 1, s_3 = 1$.

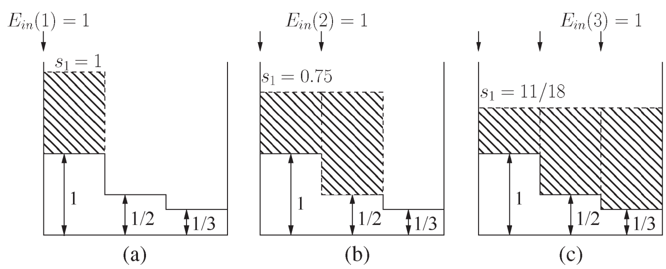

Fig. 5. Procedures to solve Example 2. (a) $s_1 = 1$. (b) $s_1 = 0.75, s_2 = 1.25$. (c) $s_1 = 11/18, s_2 = 20/18, s_3 = 23/18$.

Epoch 1 is first scanned to output RGWF(1) $= s_1 = 1$, as shown in Fig. 4(a). Now, we move to epoch 2 and apply GWF(2, 2) and output $s_2 = 1$. Check if the water level of epoch 2 $(2 + 1 = 3)$ is greater than the water level of epoch 1 $(1 + 1 = 2)$. It is true, then output the optimal solution at epoch 2: $s_1 = 1$; $s_2 = 1$, as shown in Fig. 4(b). Similarly, for epoch 3, by applying GWF(3, 3), we have $s_3 = 1$. Check the water level, it satisfies the nondecreasing condition. Hence, the algorithm outputs the completed solution, as shown in Fig. 4(c).

Example 1 is calculated without power level adjustment since the water level nondecreasing condition is satisfied for all the epochs. The channel gains (fading gains) for the three epochs are continuously deteriorating. Therefore, the harvested energy at the beginning of each epoch is fully consumed in the current epoch. In the following example, we illustrate the power level adjustment procedure. We can observe that the computed water levels also sufficiently satisfy all the other KKT conditions by the proposed algorithm.

Example 2: Suppose the fading profiles for the three epochs are $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$. At the beginning of each epoch, unit energy is harvested ($E_{in}(i) = 1, i = 1, 2, 3$). Then, the maximizing throughput problem is described by the ones that are the same as those in (31)–(33). The procedures are described in Fig. 5.

First, we scan the first epoch and RGWF(1) outputs $s_1 = 1$, as shown in Fig. 5(a). Then, move to the second epoch. By applying GWF(2, 2), it gives $s_2 = 1$. Now, check that the water level of epoch 2 is $1 + 1/2 = 1.5$ and the water level for epoch 1 is $1 + 1 = 2$. The water level nondecreasing condition is violated. Thus, the power level adjustment procedure is triggered. By applying GWF to the first two epochs, we have $\text{GWF}(1, 2) = \{s_1 = 0.75, s_2 = 1.25\}$. With this power adjustment, the new water level for both epochs is 1.75, satisfying the nondecreasing condition. The output for RGWF(2) is then $s_1 = 0.75$, $s_2 = 1.25$, as shown in Fig. 5(b).

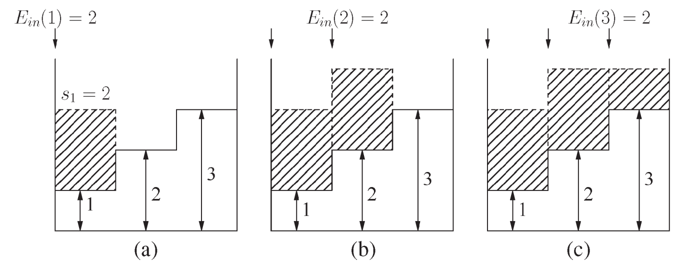

Fig. 6. Procedures to solve Example 3. (a) $s_1 = 2$. (b) $s_1 = 2$, $s_2 = 2$. (c) $s_1 = 2, s_2 = 2, s_3 = 1$.

Now, we move to epoch 3, the output of $\text{GWF}(3, 3) = s_3 = 1$. The corresponding water level for epoch 3 is 1 + 1/3, which is lower than the water level of the previous epoch (=1.75). Then, the power adjustment is triggered. The algorithm calculates the power allocation for the current epoch (epoch 3) and its previous epoch (epoch 2) to have output $\text{GWF}(2, 3) = \{s_2 = 25/24, s_3 = 29/24\}$. We move to the water level check step. The new water level of epoch 2 $((1/2) + (25/24) = 37/24)$ is lower than the water level of epoch 1 (1.75). Therefore, power adjustment needs to include epoch 1 as well. We then compute GWF(1, 3), the output is $\{s_1 = 11/18, s_2 = (11/18) + (1/2), s_3 = (11/18) + (2/3)\}$, which is the completed output for the optimal solution, as shown in Fig. 5(c).

Different from Example 1, the channel gains (fading gains) for the three epochs are continuously improving. Therefore, the harvested energy at the beginning of each epoch attempts to flow to the later epochs, leading to the uniform water level of these three epochs.

Example 3: Suppose the fading profiles for the three epochs are $a_1 = 1$, $a_2 = (1/2)$, and $a_3 = (1/3)$; the energy harvest at the beginning of each epoch is $E_{in}(1) = E_{in}(2) = E_{in}(3) = 2$. The information required for transmission is $B = 3$ bits (strictly speaking, Bbits/H$_z$). Then, the minimizing transmission completion time problem is

$$\min_{\{\{s_i\}_{i=1}^N, N\}} \quad N \tag{34}$$

$$\text{subject to} \quad \sum_{i=1}^{N} \log(1 + a_i s_i) = 3 \tag{35}$$

$$0 \le s_i \; \forall i \tag{36}$$

$$\sum_{i=1}^{l} s_i \le 2l, \; l = 1, \ldots, N. \tag{37}$$

Epoch 1 is first scanned to output RGWF(1) $= s_1 = 2$, as shown in Fig. 6(a). Since $\log(1 + 2) < B(= 3)$, we now move to epoch 2 and apply GWF(2, 2) and output $s_2 = 2$. Check if the water level of epoch 2 $(2 + 2 = 4)$ is greater than the water level of epoch 1 $(1 + 2 = 3)$. If it is true, then output the temporary optimal solution at epoch 2: $s_1 = 2$; $s_2 = 2$, as shown in Fig. 6(b). Since $\log(1 + 2) + \log(1 + 1) < B(= 3)$, we now move to epoch 3 and apply GWF(3, 3) and output $s_3 = 2$. Check if the water level of epoch 3 $(3 + 2 = 5)$ is greater than the water level of epoch 2 $(2 + 2 = 4)$. If it is true, then

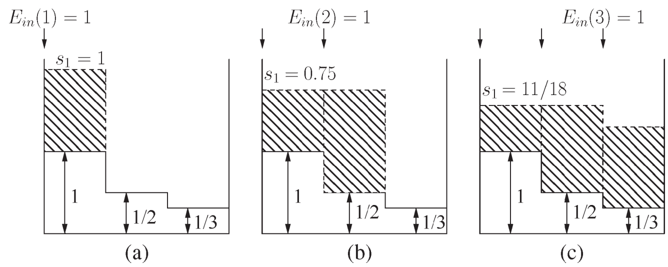

Fig. 7. Procedures to solve Example 4. (a) $s_1 = 1$. (b) $s_1 = 0.75$, $s_2 = 1.25$. (c) $s_1 = 11/18$, $s_2 = 20/18$, $s_3 = 2399/2523$.

output the temporary optimal solution at epoch 3: $s_1 = 2$; $s_2 = 2$; $s_3 = 2$. Since $\log 3 + \log 2 + \log(5/3) > B$, $B_1 = 3 - (\log 3 + \log 2) = \log(4/3)$. Then, $s_3 = (1/(1/3))(2^{\log(4/3)} - 1) = 3 \times (1/3) = 1$. Therefore, the optimal solution is $\{\{s_1^* = 2, s_2^* = 2, s_3^* = 1\}, N^* = 3\}$, and the optimal value is $N^* = 3$, as shown in Fig. 6(c).

Example 4: Suppose the fading profiles for the three epochs are $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$; the energy harvest at the beginning of each epoch is $E_{in}(1) = E_{in}(2) = E_{in}(3) = 1$. The information required for transmission is $B = \log(20)$ bits. The minimizing transmission completion time problem is described as

$$\min_{\{\{s_i\}_{i=1}^N, N\}} \quad N \tag{38}$$

$$\text{subject to} \quad \sum_{i=1}^{N} \log(1 + a_i s_i) = \log(20) \tag{39}$$

$$0 \leq s_i \; \forall i \tag{40}$$

$$\sum_{i=1}^{l} s_i \leq l, \quad l = 1, \ldots, N. \tag{41}$$

Epoch 1 is first scanned to output RGWF(1) $= s_1 = 1$, as shown in Fig. 7(a). Since $\log(1 + 1) < B (= \log(20))$, we now move to epoch 2 and apply GWF(2, 2). This gives $s_2 = 1$. Check the water level of epoch 2, which is $1 + 1/2 = 1.5$, and the water level of epoch 1, which is $1 + 1 = 2$. The water level nondecreasing condition is violated. The power level adjustment procedure is triggered. By applying GWF to the first two epochs, we have GWF$(1, 2) = \{s_1 = 0.75, s_2 = 1.25\}$. With this power adjustment, the new water level for both epochs is 1.75, satisfying the nondecreasing condition. The output for RGWF(2) is then $s_1 = 0.75$, $s_2 = 1.25$, as shown in Fig. 7(b). Since $\log(1 + 0.75) + \log(1 + 2 \times 1.25) < B$, we move to epoch 3 and apply GWF(3, 3). Similarly as in Example 2, the output of RGWF(3) is $\{s_1 = (11/18), s_2 = (11/18) + (1/2), s_3 = (11/18) + (2/3)\}$. Since $\log(1+s_1) + \log(1+2s_2) + \log(1+3s_3) > B$, $B_1 = \log 20 - \log(1+(11/18)) - \log(1+2 \times (20/18)) = \log(10 \times (18/29)^2)$. Then, $s_3 = (1/3)(2^{\log(10\times(18/29)^2)} - 1) = 2399/2523$. Therefore, the completed optimal solution is $\{\{s_1^* = (11/18), s_2^* = (20/18), s_3^* = 2399/2523\}, N^* = 3\}$, as shown in Fig. 7(c).

VII. Conclusion

For the optimal power allocation problems with energy constraints in wireless communications, we have proposed recursive algorithms to solve the problems in this paper. As a starting point, we have proposed GWF to solve the optimal power allocation problem with a sum power constraint. Then, GWF was used as a functional block to solve the problems with energy harvest in fading channels for the objective function of maximizing the data rate (RGWF) and minimizing the transmission completion time (RGWFn), respectively.

By iteratively applying GWF and comparing the computed water level with the previous epoch, RGWF outputs the optimal power allocation solutions epoch by epoch. The number of iteration is finite. RGWFn is constructed based on RGWF with additional comparison, i.e., whether the required information transmission of B bits is being achieved. Further, we have obtained and strictly proven optimality of the proposed algorithms. Numerical examples are provided to illustrate the steps to obtain the optimal solutions by the proposed algorithms.

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References

[1] J. Lei, R. Yates, and L. Greenstein, "A generic model for optimizing single-hop transmission policy of replenishable sensors," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 547–551, Feb. 2009.

[2] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1326–1336, Apr. 2010.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732–1743, Sep. 2011.

[4] C. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[5] D. P. Bertsekas, *Dynamic Programming and Optimal Control,* 4th ed. Nashua, NH, USA: Athena Scientific, 2007.

[6] D. P. Bertsekas, *Nonlinear Programming,* 2nd ed. Nashua, NH, USA: Athena Scientific, 1999.

[7] M. Gatzianas, L. Georgiadis, and L. Tassiulas, "Control of wireless networks with rechargeable batteries," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 581–593, Feb. 2010.

[8] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180–1189, Mar. 2012.

[9] J. Gong, S. Zhou, and Z. Niu, "Optimal power allocation for energy harvesting and power grid coexisting wireless communication systems," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 3040–3049, Jul. 2013.

[10] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.

[11] P. He, L. Zhao, S. Zhou, and Z. Niu, "Water-filling: A geometric approach and its application to solve generalized radio resource allocation problems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3637–3647, Jul. 2013.

[12] D. Palomar, "Practical algorithms for a family of waterfilling solutions," *IEEE Trans. Signal Process.*, vol. 53, no. 2, pp. 686–695, Feb. 2005.

[13] C. H. Papadimitriou and K. Steiglitz, *Combinatorial Optimization: Algorithms and Complexity,* Unabridged Edition. Mineola, NY, USA: Dover, 1998.

[14] D. Palomar, “A unified framework for communications through MIMO channels,” Ph.D. dissertation, Univ. Politecnica de Catalunya, Barcelona, Spain, 2003.
[15] J. Yang and S. Ulukus, “Optimal packet scheduling in an energy harvesting communication system,” *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 220–230, Jan. 2012.

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