PAPER

Adaptive Orthonormal Random Beamforming and Multi-beam Selection for Cellular Systems*

Kai ZHANG\(^{(a)}\), Student Member and Zhisheng NIU\(^{(b)}\), Member

SUMMARY Channel state information (CSI) at transmitter plays an important role for multiuser MIMO broadcast channels, but full CSI at transmitter is not available for many practical systems. Previous work has proposed orthonormal random beamforming (ORBF) \(^{[16]}\) for MIMO broadcast channels with partial channel state information (CSI) feedback, and shown that ORBF achieves the optimal sum-rate capacity for a large number of users. However, for cellular systems with moderate number of users, i.e., no more than 64, ORBF only achieves slight performance gain. Therefore, we analyze the performance of ORBF with moderate number of users and total transmit power constraint and show that ORBF scheme is more efficient under low SNR. Then we propose an adaptive ORBF scheme that selects the number of random beams for simultaneous transmission according to the average signal-to-noise ratio (SNR). Moreover, a multi-beam selection (MBS) scheme that jointly selects the number and the subset of the multiple beams is proposed to further improve the system performance for low SNR cases. The simulation results show that the proposed schemes achieve significant performance improvement when the number of users is moderate.

**key words:** – MIMO, Broadcast Channels, Random beamforming, Multi-beam selection

1. Introduction

Emerging demand for high speed wireless access has led to extensive research on multiple input multiple output (MIMO) systems. Previous work \(^{[1]}\) \(^{[2]}\) has indicated that single-user MIMO links offer remarkable spectral efficiency. However, there is increasing interest in the role of MIMO techniques in multiuser networks environment, especially in cellular downlink scenarios \(^{[3]}\). Recent results in \(^{[4]}\) \(^{[5]}\) have shown that the sum-rate capacity is achieved by dirty paper coding (DPC) \(^{[6]}\), while more recent work \(^{[7]}\) has been shown that DPC in fact achieves the capacity region of the Gaussian MIMO broadcast channel.

However, the DPC schemes, especially in the multi-user context, are difficult to implement in practical systems due to high computational burden of successive encoding and decoding. A reduced-complexity scheme referred as zero-forcing dirty paper coding (ZF-DPC) is proposed in \(^{[5]}\) \(^{[8]}\). It uses QR decomposition combined with DPC at the transmitter to optimize the sum-rate capacity and provides performance very close to the Sato bound \(^{[9]}\). A linear suboptimal strategy that can serve multiple users at a time like DPC, but with much reduced complexity, is zero-forcing beamforming (ZFBF) scheme \(^{[10]}\). If multiple antennas are used at the receiver, block diagonalization (BD) \(^{[11]}\) is a more efficient method to achieve sum-rate capacity.

While the above work achieves sum-rate capacity, it assumes perfectly known channel state information (CSI) at the transmitter. In a cellular downlink, this assumption is not reasonable. ** This is especially true, if the number of transmit antennas and the number of users are large, or the channels vary rapidly. On the other hand, using multiple transmit antennas yields no gain if no CSI is available at the transmitter. Therefore, in this paper we focus on the transmission schemes that require only partial CSI at the transmitter.

In \(^{[12]}\) \(^{[13]}\), stream scheduling is proposed for multiuser MIMO spatial multiplexing systems with partial feedback. But their work assumes that the number of receive antennas is no less than the number of transmit antennas, which is not reasonable for cellular systems. Another method named random beamforming is proposed in \(^{[14]}\) to attain coherent beamforming capacity while requiring only signal-to-noise-plus-interference ratio (SINR) as the feedback. This method uses a random beamformer and allocates resources to one user who has the best channel condition. In \(^{[15]}\), the authors extend the random beamforming method to MIMO systems. However, in their work, only one user is served at a time. An orthonormal random beamforming (ORBF) scheme with multiple random beams for MIMO broadcast channels is then presented in \(^{[16]}\). In ORBF, each user reports to the transmitter its maximum SINR along with the index number of the beam in which the SINR is maximized. Based on asymptotic analysis, \(^{[16]}\) shows that, ORBF achieves the optimal sum-rate capacity of DPC for a large number of users. But for cellular systems with moderate number of users, i.e., no more than 64, ORBF only achieves slight performance gain \(^{[17]}\). Therefore, \(^{[18]}\) proposes an improved random beamforming scheme called multi-user diversity and multiplexing (MUDAM). However, MUDAM

\(^{[a]}\) The authors are with the State Key Lab on Microwave and Digital Communications, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China. 
\(^{[b]}\) E-mail: zhangkai98@mails.tsinghua.edu.cn
\(^{[a]}\) E-mail: niuzhs@tsinghua.edu.cn

\(^{**}\) Most of cellular systems are Frequency Division Duplex (FDD) and use feedback channel to provide CSI at the transmitter.
results in long feedback delay since the users need to feed back CSI many times before all the beamforming vectors are determined. In the cellular systems with rapidly moving users, the channel states may change before all the beamforming vectors are generated and thus MUDAM is not suitable for such cases.

Therefore, this paper considers the ORBF scheme with moderate number of users and low feedback delay requirement. In most of practical wireless systems, there is hardware limitation of amplifier in transmitters, leading to limited transmit power. Moreover, in multi-cell systems, total transmit power in a base station is controlled to avoid severe interference to other cells. Therefore, besides the assumption on moderate number of users, we also assume the total transmit power constraint, which is different from [16]. Since the ORBF scheme only has partial CSI at the transmitter, it can not perfectly cancel the interference among the signals which are simultaneously transmitted to more than one user. As a result, when the average signal-to-noise ratio (SNR) increases, i.e., the system tends to be interference-limited, the performance of ORBF may degrade. That is to say, the ORBF scheme is more efficient under low average SNR. Thus we propose an adaptive ORBF scheme that selects the number of random beams for simultaneous transmission according to the average SNR. By using the proposed adaptive ORBF, the number of beams decreases as the SNR increases to maximize the sum-capacity of the systems. Moreover, since the interference among different users is relatively small for low SNR cases, we can approximately assume the interference among users is neglectable. Based on this assumption, we propose a multi-beam selection (MBS) scheme that jointly selects the number and the subset of the multiple beams to further improve the system performance for low SNR cases.

The paper is organized as follows. In Section 2 the channel model is given. After that we analyze the performance of ORBF scheme with moderate number of users and propose the adaptive ORBF scheme in Section 3. Then the multi-beam selection scheme is proposed in Section 4. Numerical examples are given in Section 5, while Section 6 contains our conclusions.

2. System Model

We consider a MIMO Gaussian broadcast channel with K receivers equipped with m_t antennas and a transmitter with m_r antennas. For a typical cellular system, the number of users is larger than the number of transmit antennas and also the number of antennas in the base station is greater than the number of antennas in the receiver. Thus we assume K > m_r > m_t. The channels are assumed to be quasi-static\(^1\) flat fading and denoted by \(H_k = \{h_{ij}(k)\}_{m_r \times m_t}\), where \(h_{ij}(k)\) is the channel gain from the \(i\)th transmit antenna to the \(j\)th receive antenna of the \(k\)th user and is assumed to be an i.i.d. zero mean complex Gaussian random variable with unitary variance. The transmitter generates random orthonormal vectors according to an isotropic distribution [19]. Firstly a random matrix \(\mathbf{T}\), whose elements are i.i.d. zero mean complex Gaussian random variables with unitary variance, is generated. Then the transmitter performs the QR factorization \(\mathbf{T} = \Phi \mathbf{R}\), where \(\Phi\) is unitary and \(\mathbf{R}\) is upper triangular. After that, the transmitter randomly selects \(m_b\) (1 ≤ \(m_b\) ≤ \(m_t\)) random orthonormal vectors \(\Phi_m\) (\(m_b\)x1) from the columns of \(\Phi\).

Given the \(m_b\) transmitted symbol \(s_m\), the received signal of the \(k\)th user is

\[
y_k = \sum_{m=1}^{m_b} H_k \Phi_m s_m + n_k, \tag{1}
\]

where \(n\) denotes the AWGN term which are modelled as i.i.d. zero mean complex Gaussian random variables with unitary variance. The long-term average SNR of the \(k\)th user is denoted by \(\rho_0\) and \(\sum_{m=1}^{m_b} E[s_m^* s_m] = \rho_0\). \(^{11}\) We assume equal power allocation to each beam, so that \(E[s_m^* s_m] = \frac{\rho_0}{m_b}\) (1 ≤ \(m\) ≤ \(m_b\)).

3. Adaptive Orthonormal Random Beamforming (ORBF) Scheme

In this section, we first analyze the sum-rate capacity of the ORBF scheme with fixed \(K\) and \(m_t\). For simplicity, we first assume \(m_r = 1\) in the derivation and then extend the conclusion to the cases with \(m_r > 1\).

Perfect CSI is assumed to be available at the receiver. Therefore, the \(k\)th receiver can compute the SINR of the \(m_b\)th beam by assuming that \(s_m\) is the desired signal and the others are interference as

\[
\text{SINR}_{km} = \frac{\|H_k \Phi_m\|^2}{m_b/\rho_0 + \sum_{i\neq m} \|H_k \Phi_i\|^2}. \tag{2}
\]

Each receiver feeds back its maximum SINR and the index which corresponds to the beam with maximum SINR. The transmitter then assigns \(s_m\) (1 ≤ \(m\) ≤ \(m_b\)) to the user with the best corresponding SINR. Therefore, the sum-rate capacity of ORBF scheme with \(m_b\) beams is

\[
R(m_b) \approx E \left[ \sum_{i=1}^{m_b} \log \left( 1 + \max \left\{ \frac{\text{SINR}_{ki}}{1} : 1 \leq k \leq K \right\} \right) \right] = m_b E \left[ \log \left( 1 + \max \left\{ \text{SINR}_{ki} : 1 \leq k \leq K \right\} \right) \right]. \tag{3}
\]

\(^{1}\)Quasi-static means that the channel is constant over a frame length and changed independently between different frames. \(^{11}\)We use \(\cdot^*\) and \(\cdot^t\) to denote the transpose and conjugate transpose operation respectively, \(\cdot^*\) to denote the conjugate of a number, and \(E[\cdot]\) to denote the expectation operator.
where the approximation is from the fact that we neglect the small probability that users may be the strongest user for more than one signal $s_i$.

Since $\Phi_m$ are orthonormal vectors, $H_k\Phi_m$ are i.i.d. zero mean complex Gaussian random variables with unitary variance over $k$ and $m$. Thus,

$$\text{SINR}_{km} = \frac{|H_k\Phi_m|^2}{m_b/\rho_0 + \sum_{i \neq m} |H_k\Phi_i|^2} = \frac{z}{m_b/\rho_0 + y},$$

where $z$ and $y$ have $\chi^2(2)$ and $\chi^2(2m_b - 2)$ distribution respectively. The probability distribution function (PDF) of SINR$_{km}$ is

$$f_s(x) = \int_0^\infty f_{s\mid y}(x\mid y)f_Y(y)dy = \frac{e^{-\frac{m_b}{\rho_0}}}{(1+x)^{m_b}} \left(m_b - 1 + \frac{m_b}{\rho_0}(1+x)\right),$$

where $f_Y(y)$ is the PDF of $y$ and $f_{s\mid y}(x\mid y)$ is the PDF of SINR$_{km}$ with given $y$. Therefore, the cumulative distribution function (CDF) of SINR$_{km}$ is

$$F_s(x) = 1 - \frac{e^{-\frac{m_b}{\rho_0}x}}{(1+x)^{m_b-1}}, \quad x \geq 0.$$  \hspace{1cm} (6)

Since SINR$_{km}$ are i.i.d. random variables over $k$, the CDF of max$_{1 \leq k \leq K}$ SINR$_{km}$ is $(F_s(x))^K$. Hence, the sum-rate capacity with $m_b$ beams is

$$R(m_b) = m_b\int_0^1 \log(1 + x) dF_s^K(x).$$ \hspace{1cm} (7)

Different from the ORBF in [16], the proposed adaptive ORBF selects the number of beams to maximize the sum-rate capacity of the system as

$$\hat{m}_b = \arg \max_{1 \leq m_b \leq m_t} R(m_b).$$ \hspace{1cm} (8)

We now begin to find out the optimal number of $m_b$ under different SNR.

Proposition 1: For fixed $K$ in low SNR cases, i.e., $\rho_0 \rightarrow \infty$, we have

$$\hat{m}_b = \arg \max_{1 \leq m_b \leq m_t} R(m_b) = m_t.$$ \hspace{1cm} (9)

Proof: When $\rho_0 \rightarrow \infty$, SINR$_{km} \approx |H_k\Phi_m|^2/\rho_0$, resulting the pdf and cdf of SINR$_{km}$ as

$$f_s(x) = \frac{m_b}{\rho_0} e^{-\frac{m_b}{\rho_0}x},$$

and

$$F_s(x) = 1 - e^{-\frac{m_b}{\rho_0}x},$$

respectively. Letting $u = F_s^K(x)$, we have

$$x = -\frac{\rho_0}{m_b} \log(1 - u^{1/K}).$$ \hspace{1cm} (12)

Substituting (10), (11), and (12) in (7), we get

$$R(m_b) = m_b\int_0^1 \log \left(1 + \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} \right) du.$$ \hspace{1cm} (13)

Therefore the differential of $R(m_b)$ over $m_b$ is

$$\frac{\partial R(m_b)}{\partial m_b} = \int_0^1 \log \left(1 + \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} \right) du - \int_0^1 \log \left(1 + \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} \right) du,$$

$$\approx \int_0^1 \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} du - \int_0^1 \frac{1}{2} \left(1 - u^{1/K} \right)^2 du,$$

$$\approx \int_0^1 \frac{1}{2} \left(1 - u^{1/K} \right)^2 du > 0,$$

where the approximation comes from $\log(1 + x) \approx x - x^2/2$ for small $x$. As a result, $R(m_b)$ increases as $m_b$ increases, leading to $\hat{m}_b = m_t$. \hspace{1cm} \Box

Although the ORBF scheme can not perfectly cancel the interference between the signals which are simultaneously transmitted to different users, in the cases with low SNR, the interference is small enough. Thus transmitting to more users simultaneously does not generate severe interference. Therefore the sum-rate capacity is maximized by using $m_t$ beams.

Proposition 2: For fixed $K$ in high SNR cases, i.e., $\rho_0 \rightarrow 0$, we have

$$\tilde{m}_b = \arg \max_{1 \leq m_b \leq m_t} R(m_b) = 1.$$ \hspace{1cm} (15)

Proof: When $\rho_0 \rightarrow 0$, we have

$$\text{SINR}_{km} \approx |H_k\Phi_m|^2/\sum_{i \neq m} |H_k\Phi_i|^2, \quad m_b > 1,$$

whose pdf and cdf are

$$f_s(x) = \frac{m_b - 1}{(1 + x)^{m_b}},$$

$$F_s(x) = 1 - \frac{1}{(1 + x)^{m_b-1}},$$

respectively. Letting $u = F_s^K(x)$, we have

$$x = \left(1 - \frac{1}{1 - u^{1/K}} \right)^{m_b-1} - 1.$$ \hspace{1cm} (19)

Substituting (17), (18) and (19) in (7), we obtain

$$R(m_b) = \frac{m_b}{m_b - 1} \int_0^1 \log \frac{1}{1 - u^{1/K}} du, \quad m_b > 1,$$

which is obviously monotonically decreasing over $m_b$. \hspace{1cm} \Box
Thus $R(2) = \max_{2 \leq m_b \leq m_t} R(m_b)$. When $m_b = 1$, we have

$$R(1) = \int_0^\infty \log(1 + x) \frac{1}{\rho_0} e^{-\frac{x}{\rho_0}} dx.$$  \hspace{1cm} (21)

For sufficiently large $\rho_0$, we can have

$$R(1) > R(2) = 2 \int_0^1 \frac{1}{u} \log\left(1 - \frac{u}{\rho_0}\right) du.$$  \hspace{1cm} (22)

Therefore $m_b = 1$ as $\rho_0 \to \infty$. \hfill \Box

Note that in the cases with high SNR, (20) shows that the sum-rate capacity of the system with $m_b \geq 2$ is not affected by $\rho_0$, i.e., the system is interference-limited. The ORBF scheme with multiple beams is not efficient in such cases. Thus it is better to use single beam at the transmitter.

For the cases with $m_r > 1$, we treat each receive antenna as an independent user. By this way, we effectively have $m_r K$ users with single antenna. Therefore, for these cases, the formulation of the problem is the same as that in the above discussion with only difference being that $K$ is replaced by $m_r K$.

4. Multi-beam Selection Scheme

As shown in the last section, ORBF is more efficient for low SNR cases. Thus the performance improvement of the proposed adaptive ORBF may be limited. Moreover, when the number of users is not large enough, subset selection among the beams can enhance the sum-rate capacity since the SINR of some beams may be too low for transmission. In this section, a multi-beam selection (MBS) scheme is proposed to further improve the system performance for such cases. Different from the adaptive ORBF, in the MBS scheme the transmitter always generates $m_r$ random beams and then selects a subset of all the $m_t$ beams based on the feedback. The receiver estimates the SINR of each beam as

$$\text{SINR}_{km} = \frac{|H_k \Phi_m|^2}{\overline{m}_b/\rho_0 + \sum_{i \neq m} |H_i \Phi_i|^2}.$$  \hspace{1cm} (23)

Denoting the maximum estimated SINR of the $k$th user as

$$\text{SINR}_{km} = \max_{1 \leq m \leq m_t} \text{SINR}_{km},$$  \hspace{1cm} (24)

the transmitter gets $\text{SINR}_{km}$ ($1 \leq k \leq K$) from the feedback of each user. The MBS scheme then jointly selects the number and the subset of the beams. Denote the selected beams and the users subset as $\mathcal{B}$ and $\mathcal{U}$ respectively, we get the SINR of the selected users as

$$\text{SINR}_{km} = \frac{|H_k \Phi_m|^2}{\tilde{m}_b/\rho_0 + \sum_{i \neq m, i \in \mathcal{B}} |H_i \Phi_i|^2}, \hspace{1cm} k \in \mathcal{U}, m \in \mathcal{B},$$  \hspace{1cm} (25)

where $\tilde{m}_b$ is the number of beams in $\mathcal{B}$. The selection of $\mathcal{B}$ and $\mathcal{U}$ is made by maximizing the sum-rate capacity as

$$\mathcal{B}, \mathcal{U} = \arg \max_{\mathcal{B} \subseteq \mathcal{M}, \mathcal{U} \subseteq \mathcal{K}} \sum_{k \in \mathcal{U}, m \in \mathcal{B}} \log \left(1 + \frac{|H_k \Phi_m|^2}{m_t/\rho_0} \right),$$  \hspace{1cm} (26)

where $\mathcal{M}$ and $\mathcal{K}$ are the sets of all the $m_r$ beams and $K$ users respectively. Unfortunately, this selection criteria is infeasible due to high complexity in practical cellular systems. However, in low SNR cases, we can approximately neglect the interference among different users and have

$$\text{SINR}_{km} \approx \frac{|H_k \Phi_m|^2}{\tilde{m}_b/\rho_0} \approx \text{SINR}_{km}' \frac{m_t}{\tilde{m}_b}.$$  \hspace{1cm} (27)

Therefore (26) can be simplified as

$$\mathcal{B} = \arg \max_{\mathcal{B} \subseteq \mathcal{M}} \sum_{m \in \mathcal{B}} \log \left(1 + \frac{m_t}{\tilde{m}_b} \max_{1 \leq k \leq K} \text{SINR}_{km}' \right),$$  \hspace{1cm} (28)

which is greatly simplified and can be executed by the transmitter based on the feedback. The flowchart of MBS is shown in Fig. 1. The transmitter needs to order the SINR of each user at the beginning and then has less than $2M_t^2$ times of multiplying operation in each loop. Therefore the transmitter totally needs $2M_t^2$ multiplying operation, which does not cause large processing delay, especially in a cellular system where the transmitter has less limitation on power consumption.

5. Numerical Examples

In this section, Monte Carlo simulations are used to compare with our analysis results. We assume $m_r = 1$ in the simulations for simplicity.

Fig. 2 plots the sum-rate capacity of the ORBF scheme with different number of random beams under low SNR ($\rho_0 = -10\, \text{dB}$). When the SNR is in middle level (10 dB), the sum-rate capacity is shown in Fig. 3. Fig. 4 demonstrates the sum-rate capacity under high SNR ($\rho_0 = 30\, \text{dB}$). The number of transmit antennas is assumed to be 10 in the simulations. The capacity increases as the number of random beams increases with $K = 60$, which matches Proposition 1 well. However, when $K = 20$, the optimal number of beams is 5 and then the capacity decreases as the number of beams increases to more than 5. The reason is that the number of users is not sufficient resulting in high probability that a user may be the strongest user for more than one signal $s_m$. Thus the approximation in (3) can not hold. In this case, due to lack of multiuser diversity gain, using more beams results in allocating the power to the beams with low gain, and therefore decreases the sum-rate capacity. Fig. 4 shows the sum-rate capacity under high SNR. The results also match the analysis of Proposition 2 well. For $m_t \geq 2$, the capacity decreases as $m_t$ increases. For the case with $K = 60$, $\rho_0$ is not sufficiently large to make the capacity of single antenna
system larger than that of the multi-beam system. But for $K = 20$, the capacity is maximized by using single antenna at the transmitter, i.e., the ORBF scheme is not suitable for the cases with high SNR. In the middle SNR case, fig. 3 shows the tradeoff between the cases of high and low SNR. The optimal number of random beams are 3 and 2 for the cases of 60 and 20 users respectively.

![Flowchart of MBS](image)

**Fig. 1** Flowchart of MBS

**Fig. 2** Sum-rate capacity with different number of random beams for $\rho_0=-10\text{dB}$

**Fig. 3** Sum-rate capacity with different number of random beams for $\rho_0=10\text{dB}$

Fig. 5 plots the sum-rate capacity of the ORBF, the proposed adaptive ORBF and MBS schemes with $m_t = 6$. We also plot the performance gain of adaptive ORBF and MBS in comparison with ORBF scheme in Fig. 6. The simulation results show that the proposed adaptive ORBF scheme achieves larger performance enhancement as SNR increases, because the optimal number of beams decreases as SNR increases but the
Fig. 4 Sum-rate capacity with different number of random beams for $\rho_0=30$dB

Fig. 5 Sum-rate capacity for ORBF and MBS schemes with $m_t=6$

ORBF scheme uses $m_t$ beams for all SNR. For low SNR, the adaptive ORBF can not improve the performance, while the MBS scheme achieves obvious improvement, which decreases as SNR increases. When SNR is more than about 10dB, the adaptive ORBF achieves better performance of the system with 10 users than the MBS scheme. The reason is that the assumption on no inter-user interference in MBS dose not hold as the SNR increases, leading to inaccuracy SINR estimation in the transmitter which reduces the performance improvement of MBS greatly. Therefore, the system can switch between MBS and adaptive ORBF based on the SNR level to achieve largest performance enhancement. Note that the gap between the curves of the ORBF and the proposed schemes become larger when there are less users in the system. That is to say, the proposed schemes work better in the systems without many simultaneously transmitting users, which is reasonable for small cell size in future cellular systems.

6. Conclusion

In this paper, we have analyzed the sum-rate capacity of a MIMO broadcast system with the ORBF scheme and moderate number of users. The analysis results have shown that it is not efficient to use multiple simultaneously transmitting beams under high SNR. We therefore have proposed an adaptive ORBF scheme, which uses different number of random beams according to the SNR level. For the cases with low SNR, we have proposed the MBS scheme to improve the sum-rate capacity. The proposed MBS scheme selects a subset of beams from all the $m_t$ beams based on the feedback information. From the simulation results, we has shown that our proposed schemes achieve great performance improvement when the number of users is in moderate level.
7. Acknowledgement

The authors would like to express sincere thanks to Hitachi R&D Headquarters for their continuous supports.

References


