Transmission Control for Wireless Networks with Inaccurate Channel Conditions: A Game-Theoretical Approach

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Abstract—In this paper, we propose a game-theoretical approach to the distributed radio resource control in wireless networks. Inaccurate channel conditions are considered by formulating the problem as a noncooperative games of incomplete information, where users are referred to as players. The operating point is the Bayesian Nash equilibrium of the game. We also study the existence and uniqueness of the equilibrium. However, the Bayesian Nash equilibrium is not Pareto efficient. Then we introduce an weight-adjusting index to player’s cost to adjust the equilibrium, which brings Pareto improvements. Numerical results show that the performance gain is significant.

I. INTRODUCTION

Radio resource control in wireless networks is essential for providing quality-of-service (QoS) guarantees due to limited system bandwidth and time-varying wireless channels. One of the major tasks of radio resource control is to control the transmission behavior of users or networks for the purpose of delay reduction or congestion avoidance. Since wireless networks are often not abundant in bandwidth, distributed resource control is widely accepted as an efficient way for reducing overhead.

With radio resource controlled in distributed manner, users compete for limited radio resources, which can be investigated by game theory. Game theory is a tool of studying the action of noncooperative competing players and their reaction to other players [1]. Recently there are some literatures investigating distributed radio resource control by game theory [2] - [6]. With the game-theoretical framework proposed in the literatures, users determines its own behavior of transmission according to its current states and its obtained knowledge about other users and the network. In [2] and [3], authors proposed a distributed power control scheme based on a N-person noncooperative game model, where users seek to maximize their own utilities in choosing transmitting power. Then the scheme is improved by introducing cost to utility functions, which brings Pareto improvements. The authors of [4] and [5] also considered power control with different forms of user utilities. In [6] the authors studied the behavior of users sharing a ALOHA medium as a noncooperative game, where users adjust their parameter of transmission behavior in an attempt to obtain their desired throughput.

However, the above literatures assume that the accurate information about each user’s channel condition is completely known by all other users. This assumption is rather impractical in wireless communication systems for it will cause intolerable overhead with the time-varying, location-dependent channels.

In this paper, we propose a distributed transmission control scheme in wireless systems with inaccurate channel conditions. Like the literatures mentioned above, We also use the concept of game theory. However, our paper differs from them in that we use the model of noncooperative games of incomplete information to formulate the condition of inaccurate channel conditions. In this model, compete users are referred to as the players who does not have complete information about each other. The operating point of the system is the Bayesian Nash equilibrium of the game. However, we find that the equilibrium is not Pareto efficient. Then we introduce an weight-adjusting index to player’s cost, which brings Pareto improvement.

The paper is organized as follows. In Section II, we present the system model, and analyze the operating point, Bayesian Nash equilibrium. Then in Section III, we introduce the weight-adjusting index. Numerical results are presented in Section IV. Finally we conclude the paper in Section V.

II. SYSTEM MODELLING

We concern a time-slotted systems. In a time slot, each user measures its channel condition. Inaccurate channel conditions are considered by formulating the problem as a noncooperative games of incomplete information, where users are referred to as players. The operating point is the Bayesian Nash equilibrium of the game. However, the above literatures assume that the accurate information about each user’s channel condition is completely known by all other users. This assumption is rather impractical in wireless communication systems for it will cause intolerable overhead with the time-varying, location-dependent channels.

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certain level of utility. If \( k \) other users transmit at the same
time (not including user \( i \) itself, i.e., \( k = 0, 1, \ldots, N \)), user \( i \)
will suffer conflicts or interferences. Let \( U_{i,k} \) represent user \( i \)’s achieved utility if it transmit together with other \( k \) users.
Since more users bring more interferences, which means less utility, we have
\[
U_{i,0} > U_{i,1} > \cdots > U_{i,N} \geq 0
\]
If user \( i \) choose to keep silent, it achieves 0 utility. For the simplicity of mathematical treatment, we assume that the
conditions of users are symmetric, i.e.,
\[
U_{i,k} = U_{j,k} = U_{k}, \quad (i, j \in \{1, 2, \ldots, N + 1\}, \quad k = 0, 1, \ldots, N)
\]
Let \( C_i \) represent the cost of user \( i \) if it choose to transmit. If it does transmit, there is no cost. Assume that \( C_i \) is relevant
to the user \( i \)’s transmit power. Due to the effects of power control, bad channel condition leads to more transmit power,
and therefore more costs. As we know, user \( i \)’s channel gain is a random variable in wireless systems, which means \( C_i \) as
function of channel gain is also a random variable. We assume that the cumulative distribution function (CDF) of
\( C_i \), represented by \( F_{C_i}(c) \) is measured in advance and is a
common knowledge to all users in the system. Without loss of generality, we assume for any \( i \) \((i = 1, 2, \ldots, N)\), \( C_i \) is
identical and independently distributed, which means
\[
F_{C_i}(c) = F_{C_i}(c) \triangleq F_C(c), \quad (i, j \in \{1, 2, \ldots, N\})
\]
With the assumption given above, the transmission control problem is formulated as a noncooperative game of incomplete
information, where users are referred to as players of the game model. The payoff profile of user \( i \), which describes the
revenue (utility minus cost) that one player can obtain corresponding to other players’ behavior, is given in Table I.

Now consider the following strategy \( c^* \): for any user \( i \), if \( C_i \leq c^* \), user \( i \) choose to transmit; if \( C_i > c^* \), it keeps silent.
Then, the expected revenue of user \( i \) if it choose to transmit is
\[
(U_0 - C_i) N \choose 0 (1 - F_{C^*})^N
\]
\[
+ (U_1 - C_i) N \choose 1 F_{C^*} (1 - F_{C^*})^{N-2}
\]
\[
+ \cdots + (U_N - C_i) N \choose N F_{C^*}^N.
\]
where \( \binom{N}{k} \) is the combination number of choosing \( k \) components from \( N \) ones, and for convenience we use \( F_{C^*} \) to
represent \( F_C(c^*) \). Note that \( F_{C^*} \) is the probability that users
choose to transmit if the threshold is \( c^* \). If user \( i \) choose to
keep silent, its expected revenue obviously is 0.

We assume users are “rational”, i.e., users pursue there
revenue-maximization [1]. Therefore, \( c^* \) should satisfy that
at \( C_i = c^* \), the expected revenue of transmitting and keeping silent is equal, which is given as
\[
\sum_{k=0}^{N} U_k \binom{N}{k} F_{C^*}^k (1 - F_{C^*})^{N-k} = c^*
\]
Eq. (1) is called equilibrium equation. The solution of the
above equation, \( c^* \), is the Bayesian Nash equilibrium of the
noncooperative game of incomplete information [1].

At the equilibrium \( c^* \), the average expected revenue of any
user \( i \) is \( J_i(c^*) \). \( J_i(c) \) is the revenue function,
\[
J_i(c) = \sum_{k=0}^{N} U_k \binom{N}{k} F_{C^*}^k (1 - F_{C^*})^{N-k} - G(c)
\]
where \( G(c) \) is given by
\[
G(c) = \int_c^\infty t f_C(x)dx
\]
and \( f_C(c) \) is the PDF of \( C_i \). Considering that \( c^* \) is the solution
to eq. (1), we have
\[
J_i(c^*) = \sum_{k=0}^{N} U_k \binom{N}{k} F_{C^*}^k (1 - F_{C^*})^{N-k} - G(c^*) = c^* F_{C^*} - G(c^*)
\]
where \( F_{C^*} = F_C(c^*) \).

III. PARETO IMPROVEMENT: WEIGHT-ADJUSTING INDEX
It can be verified that \( J_i(c^*) \) is not the optimal value of
\( J_i(x) \). In fact, the average achieved revenue of all users will
increase by the shift of the equilibrium point, which means the
Bayesian Nash equilibrium \( c^* \) is not Pareto efficient. Based on
this consideration, we introduce an weight-adjusting index
to user’s costs to adjust the equilibrium point. As a result, the
cost of users is changed to \( \alpha C_i \). In this case, if user \( i \) choose
to transmit and there are \( k \) other users transmitting at the same
time, its revenue is given by \( U_k - \alpha C_i \), and the equilibrium
equation of the adjusted game is:
\[
\sum_{k=0}^{N} U_k \binom{N}{k} F_{C^*}^k (1 - F_{C^*})^{N-k} = \alpha c^*
\]
Now the equilibrium of the game is adjusted to the solution of
eq (2), which is a function of \( \alpha \), represented by \( c^*(\alpha) \).
Therefore, the value of the new equilibrium can be controlled.
via $\alpha$. Note the aforementioned equilibrium point $c^* = c^0(1)$ ($\alpha = 1$). With the new equilibrium $c^\alpha(\alpha)$, the average achieved revenue is given by

$$J^\alpha_i(\alpha) = J_i(c^\alpha(\alpha))$$

which is a compound function of revenue function $J_i(c)$ and adjusted equilibrium $c^\alpha(\alpha)$.

We investigate the influence of $\alpha$ to the equilibrium point $c^\alpha(\alpha)$, and the most important, the average achieved revenue. It is easy to verify that $c^\alpha(\alpha)$ is a decreasing function of $\alpha$. In Section IV, we can see that in a certain rage of $\alpha > 1$, the revenue $J_i(c^\alpha(\alpha))$ as a function of $\alpha$ will increase compared with $J_i(c^*)$. Therefore, we can set such $\alpha > 1$ that the revenue of all users increases. In fact, $\alpha > 1$ leads to larger costs and smaller equilibrium point, which consequently diminishes the users’ willing of transmitting. The result is, the chance of users conflicting with each other also decreases, and the revenue is enhanced.

A further investigate shows, there exists an optimum value of the equilibrium-shifting index $\alpha_{\max}$ such that

$$\alpha_{\max} = \arg \max \alpha J^\alpha_i(\alpha)$$

i.e.,

$$J^\alpha_i(\alpha_{\max}) = \max \alpha J^\alpha_i(\alpha) \overset{\Delta}{=} J_{\max}$$

The existence of $J_{\max}$ provides the optimal solution to the transmission control, which is to set $\alpha = \alpha_{\max}$. This can be used as a reference for the distributed radio resource control in wireless networks. The corresponding optimum Bayesian Nash equilibrium $c_{\max}^\alpha = c^\alpha(\alpha_{\max})$, which is the optimal operating point of transmission control.

If exists, the numerical solution of $\alpha_{\max}$ and $J_{\max}$ could be easily found by some optimization methods with linear search (e.g., Newton’s method), or some heuristic algorithms, (e.g., Genetic algorithm) [7].

IV. NUMERICAL RESULTS

In our simulation systems, we assume that $C_1$ satisfies a Rayleigh distribution with the parameter $\sigma = 6$, i.e.,

$$f_C(x) = \begin{cases} \frac{x}{\sigma^2} \exp \left( \frac{-x^2}{2\sigma^2} \right), & (0 \leq x < +\infty) \\ 0, & (x < 0) \end{cases}$$

Also we set $U_0 = 20$, and

$$U_k = U_0 - \mu k, \quad (k = 1, 2, \cdots, N)$$

Note that $\mu$ is a parameter illustrating the differences between $U_k$. A large $\mu$ means that the value of $U_k$ descends very fast with $k$, and vice versa. Therefore $\mu$ is called descend index.

Fig. 1 present the achieved revenue $J_i^\alpha(\alpha)$ as a function of the weight-adjusting index $\alpha$. Note that in the figure the revenue of the original equilibrium is also given, by $J_i(c^*) = J^0_i(1)$, since $c^* = c^0(1)$. We see that from the figure, at a certain range of $\alpha > 1$, the achieved revenue $J_i^\alpha(\alpha)$ increases.

![Fig. 1: The achieved revenue $J_i^\alpha(\alpha)$ as a function of the weight-adjusting index $\alpha$.](image1)

![Fig. 2: Comparison of the original equilibrium $c^*$ and the optimal one of the adjusted game $c_{\max}^\alpha$ as functions of the descend index $\mu$.](image2)

![Fig. 3: Performance gain: comparison of achieved revenue $J_i(c^*)$ and $J_i^\alpha_{\max}$ as functions of the descend index $\mu$.](image3)
This phenomenon clearly shows that the original Bayesian Nash equilibrium $c^*$ is Pareto inefficient, since there exists some $\alpha > 1$ that at the corresponding equilibrium $c^0(\alpha)$, the average achieved revenues of all users will increase. Also the existence of $\alpha_{\text{max}}$ and $J_{i_{\text{max}}}$ is illustrated in this figure.

Fig. 2 shows the comparison of the original Bayesian Nash equilibrium $c^*$ and equilibrium of optimal operating point $c^0_{\text{max}}$. From this figure we observe that we $c^0_{\text{max}} < c^*$, and therefore the corresponding probability of choosing to transmit $F_{c}(c^0_{\text{max}}) < F_{c^*}$. This means that all users are operating on a lower threshold with the weight-adjusting index $\alpha = \alpha_{\text{max}}$, and therefore their willingness of transmission decreases. This will leads to less overall transmission, and therefore less confictions or interferences. As a result, the average achieved utilities of all users are enhanced.

Fig. 3 shows the performance gain of revenues with weight-adjusting index, given by $J_{i_{\text{max}}}/J_{i}(c^*)$. We can see that the improvement is obvious. Especially when $\mu$ becomes larger, the performance gain is significant.

V. CONCLUSIONS

In this paper, we propose a distributed transmission control scheme in wireless systems with inaccurate channel conditions. The problem is formulated as a noncooperative games of incomplete information. The operating point is referred as the Bayesian Nash equilibrium of the game with users referred to as players. However, the equilibrium is not Pareto efficient. Then we introduce an weight-adjusting index to player’s cost to control the equilibrium, which brings Pareto improvements. Numerical results shows that the performance gain is significant.

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