Maximum-Total-Eigenmode-Gain Based Transmit Antenna Selection in Cellular MIMO Downlink

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Abstract—In this paper, we propose the capacity-near-optimal and computation-effective Maximum Total Eigenmode Gains (MTEG) principle for the transmit antenna selection in the MIMO downlink of a cellular communication system. An Equivalent Channel Matrix (ECM) is proposed to characterize the quasi-static MIMO channel with Rayleigh fading, the co-channel interference, and the Additive White Gaussian Noise. In accordance with MTEG principle, we prove the transmit antennas should be selected according to the descending order of the norms of their corresponding column vectors in ECM. Numerical examples show that all works above were conducted in the environment of isolated MIMO channel and no co-channel interference presents.

In this paper, as a more practical situation, we consider transmit antenna selection for the MIMO channel in a cellular communication system. We only focus on the downlink situation because it is commonly expected that in future wireless communication it is more urgent to apply MIMO technique in the downlink to relieve the much heavier traffic load than in the uplink. In contrast to previous works, the contributions of this paper lie in the following two points. Firstly, instead of an isolated MIMO channel, we consider both the channel fading and co-channel interference and Additive White Gaussian Noise (AWGN) in the analysis. An Equivalent Channel Matrix (ECM) is proposed to characterize the quasi-static channel status jointly determined by the above three factors. Although some existing works had been dedicated to the capacity analysis of MIMO systems with co-channel interference [3][4], there is little similar study on transmit antenna selection when co-channel interference presents. Secondly, a Maximum Total Eigenmode Gains (MTEG) principle is proposed for the transmit antenna selection. In accordance with MTEG principle, we prove the transmit antennas should be selected according to the descending order of the norms of their corresponding column vectors in ECM. This antenna selection scheme is closed-formed and computation-effective compared to existing counterpart schemes. By numerical examples, the capacity efficiency of the proposed scheme is verified. It is shown that although the least computation complexity is required by MTEG scheme, it provides larger capacity than other existing simplified near-optimal antenna selection scheme and is more close to the optimal exhaustive search in capacity achievements.

The rest of this paper is organized as follows. We describe the system model in Section 2. In Section 3 we analyze the capacity of the co-channel-interferenced MIMO systems with transmit antenna selection. The MTEG principle and a closed-formed antenna selection scheme is presented in Section 4. Section 5 is the numerical example and we conclude our work in Section 6.

II. SYSTEM MODEL

Consider the downlink of a cellular communication system as shown in Fig.1, where there are $N_T$ transmit antennas at BS and $N_r$ receive antennas at each MS and $N_T > N_r$. The downlink is assumed to be quasi-static so that the channel can be deemed as constant in the duration of a time slot. In each time
slot, \( N_t \) out of \( N_T \) transmit antennas at BS are selected by each MS for its transmission and \( N_t \geq N_r \).

![MIMO channel with co-channel interference and transmit antenna selection](image)

For user 0, its co-channel interference presents from the transmission between BS and the other \( L \) users in the cell. At any time, the column vector of baseband signals at the \( N_r \) receive antennas of user 0 is given by

\[
y_0 = H_{0,0}x_0 + I_0,
\]

where \( H_{0,j} \) is the normalized \( N_r \times N_t \) channel matrix between the \( N_t \) selected transmit antennas of user \( j \) and the \( N_r \) receive antennas of user 0. Each element in \( H_{0,j} \) is a complex Gaussian random variable with mean 0 and variance 1 (i.e., the variance of the real and the imaginary parts are \( \frac{1}{2} \), respectively). We assume that there are a large number of scatterers in the neighborhood of the antennas and the spacing of the antennas is wide enough so that each channel response is uncorrelated. \( x_j = (x_{j1}, \ldots, x_{jN_r})^T \) is the column vector of transmitted signals of user \( j \), and \( I_0 \) is the column vector of co-channel-interference-plus-noise at user 0 and is given by

\[
I_0 = \sum_{j=1}^{L} \rho_{j} H_{0,j} x_j + n,
\]

where \( \rho_{j} \) is the co-channel interference ratio of user \( j \) and \( n = (n_1, \ldots, n_{N_r})^T \) is the vector of AWGN. The covariance matrix of \( I_0 \) is given by

\[
K_{I_0} = E(I_0I_0^H) = \sum_{j=1}^{L} \rho_{j}^2 H_{0,j}K_{x_j}H_{0,j}^H + K_n,
\]

where \( K_{x_j} \) and \( K_n \) are the covariance matrix of \( x_j \) and \( n \), respectively. \( K_n \) is a diagonal matrix with diagonal element equal to the power of background noise \( \sigma_n^2 \). On the other hand, generally speaking, the elements in \( x_j \) are not independently and identically distributed (i.i.d.) random variables thus \( K_{x_j} \) is not diagonal any more. Therefore \( I_0 \) is spatially colored and its covariance matrix \( K_{I_0} \) is non-negative definite Hermitian but not diagonal. This differs from the simple case that only AWGN presents as in [5]-[8]. In this paper, we assume the receiver has full knowledge of \( K_{I_0} \). The total transmit power of user 0 is constrained to \( P_T \), i.e., \( \text{tr}(K_{I_0}) = P_T \). Because all notations in (1) are only related to user 0, we will drop its subscript in the following analysis for the purpose of simplicity. As a result of this simplification, \( y_0, H_{0,0}, x_0 \) and \( I_0 \) are changed to \( y, H, x \) and \( I \), respectively.

### III. Capacity Analysis and Equivalent Channel Matrix (ECM)

Denote the overall channel matrix of user 0 in Fig.1 by \( \tilde{H} \). \( \tilde{H} \) is a \( N_r \times N_T \) normalized complex Gaussian matrix whose column vector \( \tilde{h}_i \) corresponds to the single-input multiple-output (SIMO) channels from the \( i \)th transmit antenna at BS to \( N_r \) receive antennas of user 0. Selecting \( N_t \) transmit antennas is just to select \( N_t \) column vectors from \( \tilde{H} \) to form \( H \) that is used for the transmission of user 0. An antenna selection is presented as a set of the labels of the selected transmit antennas, i.e., \( S = \{n_1, \ldots, n_{N_t}\} \), where \( n_i \) denotes the label of the \( i \)th selected transmit antenna at BS. The selected channel matrix \( H \) is a function of \( S \) and the \( i \)th column vector in \( H \) is given by \( h_i = \tilde{h}_{n_i} \).

Regarding the capacity of the co-channel-interfered MIMO channel, we have the following proposition.

**Proposition 1:** The mutual information presented for a \( (N_t, N_r) \) MIMO channel with channel matrix \( H \) and interference covariance matrix \( K_I \) is equivalent to that of an isolated \( (N_r, N_r) \) MIMO channel with channel matrix \( \tilde{H} \) and normalized noise, where \( H = \Lambda^{-\frac{1}{2}} V H \) and \( \Lambda^{-\frac{1}{2}} \) is a diagonal matrix and \( V \) is a unitary matrix, both of which are of dimension \( N_r \) and are completely determined by \( K_I \).

**Proof:** For a selected channel matrix \( H \), the mutual information between \( x \) and \( y \) in (1) is given by

\[
\Gamma = \log \det(I_{N_r} + H K_x H^H K_I^{-1})
\]

where \( I_M \) is the identity matrix with dimension \( M \) and we have utilized the relation \( \det(I_k + A_k B_k) = \det(I_M + B_{M \times N} A_{N \times M}) \). Because \( K_I \) is non-negative definite Hermitian, it can be decomposed into the following form

\[
K_I = U \Lambda U^H
\]

where \( U \) is an unitary matrix and \( \Lambda \) is a diagonal matrix with non-negative diagonal elements. Thus we have

\[
K_I^{-1} = (U^H)^{-1} \Lambda^{-1} U^{-1}
\]

\[
= (U^H)^H \Lambda^{-1} U^H
\]

\[
= V \Lambda^{-1} V^H
\]

where \( V = U^H \) is also an unitary matrix. Substituting (10) into (6), we have

\[
\Gamma = \log \det (I_{N_r} + H^H V H \Lambda^{-1} V H K_x)
\]
\begin{align*}
\text{Eq.}(12) \text{ is validate because } \Lambda \text{ is a diagonal matrix and all its diagonal elements are non-negative real values. Let }
H = \Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{H}.
\end{align*}

The mutual information is further written as
\begin{align*}
\Gamma = \lg \det \left( \mathbf{I}_{N_t} + \tilde{H} \tilde{H}^H \mathbf{K}_x \right).
\end{align*}

\begin{align}
(15) \text{ is exactly the mutual information of an isolated MIMO channel with normalized noise. Thus proposition 1 follows.}
\end{align}

Remarks: There are two ways to feedback the partial channel information to the transmitter. The first one is the feedback of the labels of the selected antenna set \( S \) and the second one is the feedback of the selected subset of channel matrix \( \mathbf{H} \). It is obvious the overhead introduced by the second method is larger than the first one but the channel capacity can only be reached by the second method. However, we will show later in the numerical examples that the capacity difference between these two methods is small when only a small number of transmit antennas, say \( N_t = N_r \), are selected. In this case, it is sufficient to only feedback the labels of the selected antennas to realize a large portion of capacity.

IV. MTEG Principle and Description of Proposed Antenna Selection

The problem of optimal transmit antenna selection that maximizes channel capacity is formulated by
\begin{align}
S_{\text{opt}} = \arg \max_{S} \sum_{i=1}^{N_t} \lg(1 + p_i d_i)
\end{align}
A closed-formed and accurate solution to \( (23) \) is hard to be derived because of its non-linear property. However, because the number of all possible antenna combinations is limited for a given number of total antennas \( N_T \) and a given number of selected antennas \( N_t \), a straightforward and “simple” solution to \( (23) \) is through the exhaustive search among all possible \( C_{N_t}^{N_T} \) antenna combinations for the one that provides the maximum capacity. The computation complexity of this solution growth rapidly with the increase of \( N_T \) and \( N_t \). Instead of this optimal but computation consumptive method, we propose a capacity-near-optimal but closed-formed solution to \( (23) \). This approximate solution is inspired from the eigenmode analysis of ECM.

A commonly employed realization of the MIMO channel capacity is the so called beam-forming technique [2] as shown in Fig.2, where two linear transforms \( \mathbf{W} \) and \( \mathbf{Q}^H \) are employed before the transmission of \( \mathbf{x} \) and after the receive of \( \mathbf{y} \), respectively. The matrices \( \mathbf{W} \) and \( \mathbf{Q} \) are chosen as the right and left unitary matrices respectively in the singular value decomposition (SVD) of \( \tilde{\mathbf{H}} \) such that
\begin{align}
\tilde{\mathbf{H}} = \mathbf{Q} \Pi \mathbf{W}^H,
\end{align}
where \( \Pi = \mathbf{D}^{\frac{1}{2}} \) is a diagonal matrix of real, nonnegative singular values and the square roots of the eigenvalues of \( \tilde{\mathbf{H}} \tilde{\mathbf{H}}^H \). Denote the column vectors of \( \mathbf{Q} \) and \( \mathbf{W} \) corresponding to the \( i \)th singular value by \( \mathbf{q}_i \) and \( \mathbf{w}_i \), respectively. \( (24) \) is just a compact way of writing the set of independent channels
\begin{align}
\tilde{\mathbf{H}} \mathbf{w}_i = \sqrt{d_i} \mathbf{q}_i, \quad (i = 1, \ldots, N_r)
\end{align}
Choosing one particular eigenvalue, it is noted that $w_i$ is the transmit weight factor for excitation of the singular value of $\sqrt{\mathbf{P}_i}$. A receive weight factor of $q_i^H$, a conjugate match, gives the receive power gain

$$
g_i = \left| q_i^H \sqrt{d_i} q_i \right|^2 = d_i. \quad (26)$$

This clearly shows that the equivalent MIMO channel $\tilde{H}$ is equivalent to $N_r$ independent and orthogonal single-input and single-output (SISO) channels with the eigenvalues of $\tilde{H}^H \tilde{H}$ be their respective power gains. These $N_r$ independent channels are also called as eigenmode of channel $\tilde{H}$ in the literature. Different subset of transmit antennas provides different eigenmode gains although the number of eigenmodes (or the rank of $\tilde{H}$) is $N_r$ when $N_r \geq N_t$. The basic idea of Maximum Total Eigenmode Gains principle is that we choose the transmit antenna set which provides the maximum sum of eigenmode gains among all possible antenna combinations. We must point out that this principle is not capacity-optimal because the channel capacity is determined by not only the sum of eigenvalues but also the structure of eigenvalues according to (22). Only the sum of eigenvalues is not sufficient to characterize channel capacity. However, an intuitional explanation to this principle is that a larger sum of eigenmode gains indicates a better condition for these $N_r$ eigenmodes in general thus a better SNR performance. Therefore the capacity of the antenna set with the maximum total eigenmode gains should be close to the maximum capacity provided by the optimal selection. More critically, this principle leads to the following closed-formed antenna selection algorithm.

**Proposition 2**: Ordering transmit antennas by the descending order of the Frobenius norm of the corresponding equivalent channel vector, i.e., $\|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_1\|_F \geq \|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_2\|_F \geq \ldots \geq \|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{N_r}\|_F$, then the antenna selection $\mathbf{H}^o = (\mathbf{h}_1 \ldots \mathbf{h}_{N_t})$ provides the maximum total eigenmode gains among all antenna selections for a given $N_t$.

**Proof**: Let $\mathbf{H}$ be the channel matrix of an arbitrary antenna selection, i.e., $\mathbf{H} = (\mathbf{h}_{n_1} \ldots \mathbf{h}_{n_{N_t}})$. Let $d_1, \ldots, d_{N_r}$ be the eigenvalues of $\tilde{H}^H \tilde{H}$. The product of $\tilde{H}^H \tilde{H}$ is given by

$$
\tilde{H}^H \tilde{H} = \begin{bmatrix}
(\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_1})^H \\
(\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_2})^H \\
\vdots \\
(\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_{N_t}})^H \\
\end{bmatrix} 
\begin{bmatrix}
\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_1} \\
\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_2} \\
\vdots \\
\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_{N_t}} \\
\end{bmatrix}^T
= \begin{bmatrix}
\|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_1}\|_F^2 \\
\|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_2}\|_F^2 \\
\vdots \\
\|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{n_{N_t}}\|_F^2 \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{h}_{n_1}^H \mathbf{K}_1^{-1} \mathbf{h}_{n_{N_t}} \\
\mathbf{h}_{n_2}^H \mathbf{K}_1^{-1} \mathbf{h}_{n_{N_t}} \\
\vdots \\
\mathbf{h}_{n_{N_t}}^H \mathbf{K}_1^{-1} \mathbf{h}_{n_{N_t}} \\
\end{bmatrix}
\geq \sum_{i=1}^{N_r} d_i \quad \text{tr} \left( \tilde{H}^H \tilde{H} \right) \quad (29)
$$

Then the sum of the eigenmode gains is given by

$$
\sum_{i=1}^{N_r} d_i = \text{tr} \left( \tilde{H}^H \tilde{H} \right) \quad (29)
$$

Let the eigenvalues of $\tilde{H}^o = \tilde{H}^H \tilde{H}^o$ be $d_1, \ldots, d_{N_r}$. Following the similar process of (30), the sum of the eigenmode gains of the effective channel matrix $\tilde{H}^o$ is given by

$$
\sum_{i=1}^{N_r} d_i = \sum_{i=1}^{N_r} \|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{i}\|_F^2. \quad (31)
$$

Because $\|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_1\|_F \geq \|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_2\|_F \geq \ldots \geq \|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_{N_r}\|_F$, thus we have

$$
\sum_{i=1}^{N_r} d_i \geq \sum_{i=1}^{N_r} d_i. \quad (32)
$$

And then Proposition 2 follows.

**Proposition 2** provides a computational effective and closed-formed antenna selection scheme. According to Proposition 2, the receiver only need to compute the Frobenius norm of each equivalent channel vector $\|\Lambda^{-\frac{1}{2}} \mathbf{V} \mathbf{h}_i\|_F$, the $N_t$ transmit antennas with the $N_t$ largest Frobenius norms are selected for the transmission of user 0. The selected antenna set owns the largest sum of eigenmode gains among all antenna combinations and is expected to achieve a near-optimal channel capacity. In [8], a simplified and capacity near-optimal Sandhu’s scheme was also proposed for the transmit antenna selection. Here, for the purpose of comparison, we also outline the Sandhu’s scheme as follows:

This selection scheme induces a $N_T \times N_T$ covariance matrix $\mathbf{K}$ that has $N_t$ diagonal elements (corresponding to selected antennas) equal to $\frac{P_T}{N_T}$ and zeros elsewhere. The covariance matrix $\tilde{\mathbf{K}}$ obtained by approximating $\mathbf{K}_x$ as described in (18) when all $N_T$ transmit antennas are used is the closest to $\mathbf{K}_x$ over all selection matrices in terms of Frobenius norm, i.e.

$$
\tilde{\mathbf{K}} = \arg\min_{\mathbf{K}} \|\mathbf{K} - \mathbf{K}_x\|_F.
$$

It is clear that the computation complexity of MTEG scheme is much simpler than the exhaustive search scheme and it is also simpler than the Sandhu’s near-optimal scheme. The capacity performance of the new scheme is examined by numerical examples in the next section and it shows that although the smallest computation is required by the new scheme, it provides larger capacity than Sandhu’s scheme and thus its performance is more close to the optimal exhaustive search.

V. Numerical Examples

The capacity of a MIMO system is a function of the channel matrix. Thus the capacity is a random variable with some distribution. Monte Carlo simulations were performed for a wireless system with 10 transmit and 2 receive antennas. Three transmit antenna selection schemes are simulated: the exhaustive search, the Sandhu’s scheme, and the proposed MTEG scheme.
Because this numerical example is for the comparison of capacity achievements under different schemes, we did not simulate the colored co-channel interference for simplicity thus only normalized AWGN was presented. This simplification does not influence their relative performance because all three schemes are compared under the same environment. Under each antenna selection scheme, the labels of selected antennas and the selected subset of MIMO channel matrix are fed back to the transmitter for equal power transmission and water-filling plus beamforming transmission, respectively.

The cumulative density functions of capacity and the average capacity as functions of signal-to-noise-ratio (SNR) under different antenna selection schemes are plotted in Figs.3-4, respectively. In these figures, 2 out of 10 transmit antennas are selected in each time slot for transmission. The results of no CSI feedback (i.e., 10 transmit antennas are all used for transmission and the transmission powers are distributed equally across them) are also presented for comparison. It is shown that the channel capacity can be considerably improved by transmit antenna selection. This is due to the fact that by transmit antenna selection, the transmission power is focused on those antennas with better channel conditions thus the received SNR is improved, which in turn is translated into the improvement of capacity. The superiority of MTEG scheme is clearly verified by comparing the curves of the three schemes: although MTEG scheme requires the least computation complexity, its capacity contribution is larger than that of the Sandhu’s scheme and is very close to the optimal capacity provided by the exhaustive search. This advantage is explained by noting that, according to (22), the total eigenmode gain is more directly and closely related to the channel capacity than the covariance matrix of transmit symbols. Thus the a selection criteria based on the maximum total eigenmode gains should be more relevant than its competitor in the sense of capacity improvement. Another interesting fact presented in Figs.3-4 is that the capacity difference between equal power transmission and water-filling plus beamforming transmission is very limited when only two transmit antennas are selected. This is explained by noting that the difference between the two eigenvalues of the selected (2,2) MIMO system is not substantial, thus the power allocations under water-filling principle differs only in a limited degree from the equal power allocation. This fact suggests that it may be sufficient to feedback only the labels of the selected antennas to realize a large portion of capacity when the minimum number (i.e., $N_t = N_r$) of transmit antennas are selected.

VI. CONCLUSIONS

In this paper, we have presented the maximum total eigenmode gains principle for the transmit antenna selection in a co-channel interfered MIMO system. Based on this principle, we have proved that the transmit antennas should be selected according to the descending order of the Frobenius norms of their corresponding vectors in the equivalent channel matrix. This is a closed-formed antenna selection and is highly computation-effective compared to previous schemes. By numerical examples, we have verified the superiority of the proposed MTEG scheme: although it requires the least computation complexity, it provides larger capacity than other simplified near-optimal competitor and is more close to the optimal exhaustive search scheme in capacity achievements.

REFERENCES