Abstract—In this paper, we study the influence of decoding order on the capacity of multimedia DS-CDMA systems with imperfect successive interference cancellation. In contrast to previous studies, cancellation errors are assumed to be different for different users in this work. For any given decoding order, we derive the necessary power allocation that guarantee the QoS of the multimedia traffic. Based on this result, we prove that instead of by the descending order of data rate as suggested in some literature, the system capacity is maximized by decoding users according to the ascending order of cancellation errors. We also prove that this capacity-optimal decoding order makes total residual interference minimum at the same time. Our results are verified by numerical example.

I. INTRODUCTION

Code Division Multiple Access (CDMA) system is characterized as being interference limited, and the multiple access interference (MAI) is the most important factor that degrades its capacity. The successive interference cancellation (SIC)[1] is a sub-optimal yet simple technique proposed to mitigate MAI and enhance uplink capacity in CDMA networks. By SIC, the decoded signals are successively subtracted from the composite signal at base station (BS). Thus the subsequently decoded users will experience reduced interference. In order to perform SIC, signal parameters such as amplitude and phase have to be estimated[5] when the user’s signal is decoded. Due to the fading of wireless channel, there exist errors in the parameter estimation so that only part of the decoded signal power can be mitigated from the composite power. The remaining part, i.e. the cancellation error or namely residual interference, will influence the decoding of subsequent users. This is called imperfect SIC[6], and on the contrary it is referred to as perfect SIC[7] if no cancellation error exists.

A performance-sensitive issue in SIC is the decoding order of users. For a system with $K$ users, the total number of possible decoding orders is $K!$, each of which may result in different system performance. So far, a lot of works have been dedicated to the optimal decoding order of SIC. For CDMA systems supporting only single traffic class, a commonly suggested method is to rank users according to the descending order of their received powers[1]-[4]. This ordering is done for two reasons. First, the strongest signals are generally the most reliable and thus can be detected in the presence of more interference. Second, the strongest signals on average cause the most interference and thus removing the effects of these signals has the greatest benefit. On the other side, the case is more complicated for multimedia CDMA systems. Some researchers[8] have suggested the so called rate-ordering in multi-rate CDMA networks, by which users are decoded in descending order of data rate. The reason behind this method lies in that generally the user with higher transmission rate presents larger interference to other users, thus decoding such user first will bring more benefit. In this sense, the rate-ordering is similar to the power-ordering suggested in [1]-[4]. Another decoding order, ranking users in descending order of channel gains, was proposed in [7] to minimize total transmission powers for multimedia CDMA networks with the assumption of perfect SIC. In [6], the same problem was considered for imperfect SIC. The main drawback of [6] is its assumption that the cancellation errors of all users are equal. In fact, because the accuracy of signal parameter estimation depends largely on the channel state, the cancellation error varies from user to user. Some methods of channel estimation are given in [9] and [10].

On the other hand, in order to utilize system resource efficiently, it is important to find an optimal decoding order for imperfect SIC, by which maximum system capacity can be achieved. In this work, we study the capacity-optimal decoding order for a multimedia CDMA system with imperfect SIC, whose uplink capacity is constrained by a pre-defined maximum total received power threshold. Pilot signals are employed in the uplink for channel estimation. The users’ cancellation errors are diverse and derived according to the channel estimation errors. Instead of by the commonly suggested rate-ordering, in this paper we prove that by decoding users according to the ascending order of cancellation errors, system capacity is maximized or equivalently, total received power is minimized. In addition, we also prove that the same decoding order minimizes total residual interference at the same time. Our work indicates the inadequacy of ranking users only according to their power levels (or data rates) by suggesting that a more comprehensive parameter, the cancellation error, should be considered in the ordering in order to derive the maximum system capacity.

The rest of this paper is organized as follows. In Section 2, we describe the system model and derive the power allocation under imperfect interference cancellation. In Section 3, we analyze the optimal decoding order which maximizes system capacity. Section 4 is the numerical example and we conclude our work in Section 5.

Capacity Optimization by Using Cancellation-Error-Ascending Decoding Order in Multimedia CDMA Networks with Imperfect Successive Interference Cancellation

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II. System Model and Power Allocation

We consider a single cell pilot-assisted DS-CDMA system with multimedia traffic, in which pilot symbols are inserted to the data frame of each user for the purpose of channel estimation. All users’ signals are spread over the whole bandwidth with respective spreading factors. The total bandwidth is 2 Hz, and the single-sided power density of background AWGN is $N_0$ Watts/Hz. We only consider the uplink, since it is the main limitation on system capacity in cellular system. Let $K$ be the number of users in the cell. SIC is applied to the uplink and downlink, and QoS requirement of the $i$th user is considered. 

To study in [9], the error of channel estimation is mainly due to the BS, i.e. 

$$E_i = \frac{\epsilon_i G_i}{1 + G_i},$$

where we ignore the power control. When power control is presented in the system, all these factors are of no relation with the decoding order. Thus the cancellation error of a user can be seen as an independent parameter from its decoding order.

For user $i$, its received base-band signal at BS is given by

$$s_i(t) = g_i(t - \tau_i)e^{j\phi_i(t - \tau_i)} \sqrt{E_i}b_i(t - \tau_i)a_i(t - \tau_i),$$

where $g_i(t)$ and $\phi_i(t)$ are the amplitude and the phase of channel gain, respectively, and are acquired from the detecting of pilot symbols. $E_i$ is the transmission power, $b_i(t)$ is the data bit and $a_i(t)$ is the spread sequence of user $i$. After user $i$ is decoded, its signal is regenerated according to the channel gain estimation and then subtracted from the received composite power. The resulted residual interference of user $i$ after the subtraction is given by

$$Q_i = \left| g_i e^{j\phi_i} \sqrt{E_i} - g_i e^{j\phi_i} \sqrt{E_i} \right|^2$$

$$= E_i \left[ g_i^2 + \hat{g_i}^2 - 2g_i\hat{g_i} \cos (\phi_i - \phi_i) \right],$$

where we ignore the $(t - \tau_i)$ for simplicity. $g_i$ and $\phi_i$ are the amplitude and the phase of channel gain estimated such as in [10]. The cancellation error of user $i$ is just given by

$$\epsilon_i = E \left\{ \frac{Q_i}{g_i^2 E_i} \right\}$$

$$= E \left\{ 1 + \left( \frac{\hat{g_i}}{g_i} \right)^2 - 2 \left( \frac{\hat{g_i}}{g_i} \right) \cos (\phi_i - \phi_i) \right\}.$$  

In the practical implementation, the expectation operation can be replaced by the averaging of historical samples. According to the study in [9], the error of channel estimation is mainly a function of user’s signal-to-interference ratio ($E_b/I_0$), the Doppler frequency (determined by user’s velocity), and the pilot symbol scheme. When power control is presented in the system, all these factors are of no relation with the decoding order of the user. Thus the cancellation error of a user can be seen as an independent parameter from its decoding order.

Considering imperfect cancellation, in order to guarantee the QoS requirement of the $i$th user, the received powers must satisfy

$$\gamma_i = \frac{W}{R_i \sum_{j=1}^{i-1} \epsilon_j P_j + \sum_{j=i+1}^{K} P_j + N_0 W}, \quad (i = 1, \ldots, K)$$

where the first sum item in the denominator represents the total residual interference after the interference mitigation of the previous $i - 1$ users, and the second sum item represents the interference from the remaining $K - i$ users. Let $G_i = \frac{\epsilon_i G_i}{1 + G_i}$, for all the users in the system we yield the following matrix equation according to (5)

$$\begin{pmatrix} 1 & -G_1 & -G_1 & \ldots & -G_1 \\ -\epsilon_1 G_2 & 1 & -G_2 & \ldots & -G_2 \\ -\epsilon_1 G_3 & -\epsilon_2 G_3 & 1 & \ldots & -G_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\epsilon_1 G_K & -\epsilon_2 G_K & -\epsilon_3 G_K & \ldots & 1 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_K \end{pmatrix} = \begin{pmatrix} N_0 W G_1 \\ N_0 W G_2 \\ N_0 W G_3 \\ \vdots \\ N_0 W G_K \end{pmatrix}.$$  

(6)

By some tedious but straight-forward mathematical manipulation, we derive the following closed-form solution

$$P_i = \frac{N_0 W G_1}{1 - G_1 \sum_{j=2}^{K} \prod_{j=2}^{i} \left( \frac{G_j}{1 + G_j} \right) \left( \epsilon_{j-1} + \frac{1}{\epsilon_{j-1}} \right)},$$

(7)

$$P_i = \frac{P_1 \prod_{j=2}^{i} \left( \frac{G_j}{1 + G_j} \right) \left( \epsilon_{j-1} + \frac{1}{\epsilon_{j-1}} \right)}{1 - \sum_{j=2}^{K} \prod_{j=2}^{i} \left( \frac{G_j}{1 + G_j} \right) \left( \epsilon_{j-1} + \frac{1}{\epsilon_{j-1}} \right)}, \quad (i = 2, \ldots, K)$$

(8)

For any given order of user decoding, (7) and (8) are the power allocation which are necessary to satisfy all users’ QoS requirements under imperfect SIC. A similar study was conducted in [7], where the power allocation for multimedia users was derived with the assumption of perfect SIC. We will compare these two power allocation schemes in the numerical example to show that in a practical system with imperfect SIC, great degradation of user performance is resulted from the simplification by assuming perfect SIC, and therefore such assumption is far from adequate.

III. Optimal Decoding Order Analysis

Denote by $P_T$ the total received power at BS, i.e. $P_T = \sum_{i=1}^{K} P_i$. From the first equation in (6), after some simple mathematical manipulation $P_T$ is given by

$$P_T = P_1 \frac{1 + G_1}{G_1} - N_0 W.$$  

(9)

Substituting (7) into (9), $P_T$ is given by

$$P_T = \frac{N_0 W}{\frac{1}{1 + G_1} - \frac{G_1}{1 + G_1} \sum_{j=2}^{K} \prod_{j=2}^{i} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{\epsilon_{j-1}} \right)} - N_0 W.$$  

(10)

From the denominator of (10), it is indicated that different user decoding orders may result in different total received powers. For the existence of feasible power allocation, the denominator in (10) must be positive. It is easy to verify that the denominator decreases with the increase of user number. Therefore, under the power constraint $\tau$, in order to attain the maximum capacity, the user decoding order should be arranged to
maximize the denominator or equivalently to minimize total received power. For the given \( K \) users, there are total \( K! \) decoding orders. Our objective of maximizing system capacity is just to select the one which minimizes \( P_T \) in (10).

**Proposition 1:** For the given \( K \) users, the optimal user decoding order that minimizes total received power is the ascending order of the cancellation error, i.e. \( \epsilon_1 \leq \epsilon_2 \leq \ldots \leq \epsilon_K \).

The proof of Proposition 1 depends on the following two lemmas.

**Lemma 1:** Given the first decoded user, the conditional total received power is minimized by decoding remaining users according to the ascending order of their cancellation errors.

**Proof:** Because the first decoding user is given, we only need to consider the second term (i.e. the sum term) in the denominator of (10). Denote \( S_i = \prod_{j=2}^{i} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{G_{j-1}} \right) \), \( (i = 2, \ldots, K) \). Let A and B be the \( i \)th \((i \neq 1)\) user and the \( i+1 \)st user in the decoding, respectively. Then

\[
S_A = \left[ \prod_{j=2}^{i-1} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{G_{j-1}} \right) \right] \times \frac{G_A}{1 + G_A} \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right), \tag{11}
\]

\[
S_B = \left[ \prod_{j=2}^{i-1} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{G_{j-1}} \right) \right] \frac{G_B}{1 + G_B} \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right) \times \frac{G_A}{1 + G_A} \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right) \tag{12}
\]

If the order of A and B is switched, i.e. B is the \( i \)th user and A is the \( i+1 \)st user, then

\[
S_B' = \left[ \prod_{j=2}^{i-1} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{G_{j-1}} \right) \right] \times \frac{G_B}{1 + G_B} \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right), \tag{13}
\]

\[
S_A' = \left[ \prod_{j=2}^{i-1} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{G_{j-1}} \right) \right] \frac{G_B}{1 + G_B} \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right) \times \frac{G_A}{1 + G_A} \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right) \tag{14}
\]

It can be shown that \( S_j = S'_j \) for all \( (j = 2, \ldots, i-1, i + 2, \ldots, K) \). Thus

\[
\sum_{j=2}^{K} S_j - \sum_{j=2}^{K} S_j' = (S_A + S_B) - (S_A' + S_B') \]

\[
= \frac{G_AG_B(\epsilon_A - \epsilon_B)}{(1 + G_A)(1 + G_B)} \times \left[ \prod_{j=2}^{i-1} \frac{G_j}{1 + G_j} \left( \epsilon_{j-1} + \frac{1}{G_{j-1}} \right) \right] \times \left( \epsilon_{i-1} + \frac{1}{G_{i-1}} \right). \tag{15}
\]

Thus the total received power of the decoding order, A before B, is smaller if \( \epsilon_A < \epsilon_B \). Given the first decoding user, we apply this process recursively for any pair of users after the first user, then the conditional total received power is minimized when \( \epsilon_2 \leq \ldots \leq \epsilon_K \). Then Lemma 1 holds.

**Lemma 2:** Given the decoding order from the 3rd user to the last user, the conditional total received power is smaller if \( \epsilon_1 < \epsilon_2 \).

**Proof:** Let A be the first decoding user and B be the second decoding user. Then

\[
S_B = \frac{G_B}{1 + G_B} \left( \epsilon_A + \frac{1}{G_A} \right). \tag{16}
\]

Then we switch the decoding order of A and B and derive

\[
S_A' = \frac{G_A}{1 + G_A} \left( \epsilon_B + \frac{1}{G_B} \right). \tag{17}
\]

It can be shown that

\[
G_A \frac{1}{1 + G_A} \sum_{j=2}^{K} S_j = G_B \frac{1}{1 + G_B} \sum_{j=2}^{K} S'_j, \quad (j = 3, \ldots, K) \tag{18}
\]

Thus

\[
\frac{G_A}{1 + G_A} \sum_{j=2}^{K} S_j = \frac{G_B}{1 + G_B} \sum_{j=2}^{K} S'_j \]

\[
= \frac{G_A G_B (\epsilon_B - \epsilon_A)}{(1 + G_A)(1 + G_B)} \tag{19}
\]

Thus the total received power of the decoding order, A is the first and B is the second, is smaller if \( \epsilon_A < \epsilon_B \). And then Lemma 2 follows.

By applying Lemma 1 and Lemma 2 recursively, the proof of Proposition 1 is obvious and straight-forward. Because this optimal decoding order is determined only by the cancellation errors in SIC, it is named as \( \epsilon \)-ordering.

Denote by \( Q_T \) the total residual interference at BS, i.e. \( Q_T = \sum_{i=1}^{K} \epsilon_i P_i \). For any given decoding order, another power allocation equivalent to (7) and (8) can be derived from (6) as following

\[
P_K = \frac{N_0 W}{F_K - \sum_{i=1}^{K-1} \epsilon_i \prod_{j=i}^{K-1} \frac{1 + F_{j+1}}{\gamma_j + F_j}}, \tag{20}
\]

\[
P_i = P_K \prod_{j=i}^{K-1} \frac{1 + F_{j+1}}{\epsilon_j + F_j}, \quad (i = 1, \ldots, K - 1) \tag{21}
\]

where \( F_i \) is the inverse of \( G_i \), i.e. \( F_i = \frac{W}{\pi_i^2} \). From the last equation in (6), \( Q_T \) is derived as

\[
Q_T = P_K (F_K + \epsilon_K) - N_0 W \tag{22}
\]

\[
= \frac{F_K}{F_K + \epsilon_K} - \frac{1}{F_K + \epsilon_K} \sum_{i=1}^{K-1} \epsilon_i \prod_{j=i}^{K-1} \frac{1 + F_{j+1}}{\gamma_j + F_j} - N_0 W \tag{23}
\]
Proposition 2: The decoding order which makes total received power minimum must make total residual interference minimum at the same time, and vice versa.

The proof of Proposition 2 is related to the following two lemmas.

Lemma 3: Given the last decoded user, the conditional total residual interference is minimized by decoding the first user to the $K - 1$st user according to the ascending order of their cancellation errors.

Lemma 4: Given the decoding order from the first user to the $K - 2$nd user, the conditional total residual interference is smaller if $\epsilon_{K-1} < \epsilon_K$.

Both Lemma 3 and Lemma 4 can be proved in the similar manner to the proof of Lemma 1 and Lemma 2, respectively. By applying Lemma 3 and Lemma 4 recursively, the total residual interference is minimized when the users are decoded according to the ascending order of their cancellation errors, which is the same order that minimizes total received power according to Proposition 1. Thus Proposition 2 is proved.

Remarks: At first glance of Propositions 1 and 2, it is surprising that the optimal order does not depend on the relative power levels among different users. This misunderstanding can be clarified by noting that, in fact, the cancellation error of each user is highly related to its received SIR. Thus the received power level does have its influence on the optimal order. Besides SIR, the other factors that influence cancellation error are the Doppler frequency and the pilot symbol scheme, as stated in [9]. Our work indicates the inadequacy of ranking users only according to their power levels (or rates) by suggesting that a more comprehensive parameter, the cancellation error, should be considered in the ordering in order to derive the maximum system capacity.

IV. NUMERICAL EXAMPLE

We consider a DS-CDMA system with three traffic classes, whose rate requirements are 16kb/s, 64kb/s, 128kb/s and $E_b/I_0$ requirements are 4, 6, 8, respectively. Denote the maximum cancellation error of traffic class $i$ by $\epsilon_{maxi}$ and the user number of traffic class $i$ by $K_i$. Assume the cancellation error profile of traffic class $i$ to be $(\frac{\epsilon_{max1}}{K_1}, \frac{\epsilon_{max2}}{K_2}, \ldots, \frac{\epsilon_{maxi}}{K_i})$. We fix $K_1$ and $K_2$ to be 3, and $\epsilon_{max1}$ and $\epsilon_{max2}$ to be 0.1. $K_2$ and $\epsilon_{max2}$ are varied as parameters. The total bandwidth is $W = 5$MHz and power density of AWGN is $N_0 = 10^{-3}$W/Hz.

We compare the performance of $\epsilon$-ordering to that of rate-ordering[8]. By rate-ordering scheme, the user with higher data rate will be decoded with higher priority. Because the user with higher transmission rate usually presents larger interference to other users in the system, this scheme is similar to the power-ordering scheme commonly suggested in single traffic CDMA systems[1]-[4]. Furthermore, in order to highlight the effect of $\epsilon$-ordering, in rate-ordering when two or more users are of the same rates, the user with largest cancellation error is decoded first. We should note that this action does not offend the principle of the rate-ordering scheme, which does not distinguish the users with the same rate but with the different cancellation errors.

In Fig.1, we plot the maximum number of admissible type 2 users as functions of $\epsilon_{max2}$ with respect to different composite received power constraint. It shows that with the increase of cancellation error, system capacity degrades. However, great capacity gain is attained by $\epsilon$-ordering over rate-ordering. This is due to the fact that the total received power is much smaller by $\epsilon$-ordering than it is by rate-ordering (see Figs.2 and 3), so that more users can be admitted by $\epsilon$-ordering under the same power constraint. In addition, we see that, even when $\tau = \infty$, the capacity is limited, since the required power levels increase infinity as the user number approaches towards the capacity limits (see Fig.2).

The total received power and the total residual interference are plotted as functions of number of type 2 users and $\epsilon_{max2}$ in Figs.2 and 3, respectively. In Fig.2, $\epsilon_{max2}$ is set to be 0.1 and in Fig.3 $K_2$ is set to be 15. Figs.2-3 show that the total received power and the total residual interference increase with the increase of user number and cancellation error. However, smaller received power is required and smaller residual interference is left when the $\epsilon$-ordering is employed.

We study the degradation of user performance resulted from the assumption of perfect SIC in Fig.4. The power allocation derived in [7] is employed for the comparison. $E_b/I_0$ of all three traffic classes are plotted as functions of type 2 user number under the power allocations with and without perfect SIC assumption, respectively. It shows that under the assumption of perfect SIC (the result in [7]), no user’s QoS requirement can be guaranteed: all users’ $E_b/I_0$ decrease with the increase of user number. The deviation from the targeted value becomes...
This optimal decoding order also minimizes the total received power and the total residual interference under imperfect SIC. By numerical example the validity of our analysis was verified. Our work indicates the inadequacy of ranking users only according to their power levels (or data rates) by suggesting that a more comprehensive parameter, the cancellation error, should be considered in the ordering in order to derive the maximum system capacity.

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V. CONCLUSIONS

We have proved that the capacity of multimedia DS-CDMA system employing imperfect SIC is maximized by decoding users according to the ascending order of cancellation errors. Very large when the user number is large. This is explained by the fact that the residual interference of a user will have impact on the decoding of all subsequent users, thus the cumulative cancellation error will increase with the increase of user number. Fig.4 indicates neglecting the cancellation error in SIC is far from adequate in providing QoS guarantees for multimedia traffic.