MIMO Broadcast Transmission with Outdated Channel State Information

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Abstract

The performance of multiuser MIMO downlink systems with block diagonalization (BD) relies on the channel state information (CSI) at the transmitter to a great extent. For time division duplex TDD systems, the transmitter estimates the CSI while receiving data at current time slot and then uses the CSI to transmit at the next time slot. When the wireless channel is time-varying, the CSI for transmission is imperfect due to the time delay between the estimation of the channel and the transmission of the data and severely degrades the system performance. In this paper, we propose a linear method to suppress the interferences among users and data streams caused by imperfect CSI at transmitter. The transmitter first sends pilot signals through a linear spatial precoding matrix so as to make possible that the receiver can estimate the interference, and then the receiver exploits a linear prefilter to suppress the interference. The numerical results show that the proposed schemes achieve obvious performance enhancement in comparison to the scheme assuming perfect CSI at the transmitter.

1 Introduction

Due to the rapid expansion of wireless communication systems, the interest in multiple-input-multiple-output (MIMO) systems is growing extensively. Previous work [4][13] has indicated remarkable spectral efficiency of single-user MIMO links. However, there is increasing interest in multiuser MIMO systems, especially in downlink broadcast scenarios [11]. It has been shown that Dirty paper coding (DPC) [3] achieves the sum-rate capacity of MIMO broadcast channel [14][2], and even the capacity region [7]. However, because of high computational burden of successive encoding and decoding, the DPC schemes are difficult to implement in practical systems. A suboptimal strategy that can serve multiple users at a time like DPC, but with much reduced complexity, is block diagonalization (BD) [10], in which the base station (BS) performs linear spatial precoding enforcing a zero inter-user interference constraint. In BD, data streams from different users are multiplexed in the spatial domain.

The above work is based on the assumption that channel state information (CSI) is perfectly known at the transmitter. However, in many applications, this assumption is not reasonable, especially if the number of transmit antennas and the number of users are large, or the channels vary rapidly. In Frequency Division Duplex (FDD) systems, the CSI at the transmitter is sent by the receiver through a feedback channel, which is usually bandwidth limited. Therefore the CSI at transmitter is usually partially fed back. There is some recent work on multiuser MIMO systems with partial CSI feedback [8],[16],[12]. However, this paper focuses on the Time Division Duplex (TDD) systems, which becomes more and more important nowadays. For TDD systems, the uplink and downlink channels are time-multiplexed on the same physical channel. The transmitter estimates the CSI while receiving data at current time slot and then uses the CSI to transmit at the next time slot. When the wireless channel is time-varying, the CSI for transmission is imperfect due to the time delay between the estimation of the channel and the transmission of the data. For multiuser MIMO systems with BD, outdated CSI at transmitter results in not only self-interference of different data streams but also interference among users, and severely degrades the performance especially in the cases with highly moving users or long delay.

Recent work [15][5] has considered imperfect CSI at the transmitter for single user MIMO systems. Self-interference of different data streams caused by imperfect CSI degrades the performance of MIMO systems. In [15], the authors use perturbation analysis to evaluate the self-interference and propose a power allocation strategy to achieve specified performance. A new architecture is proposed in [5] to counter the effect of imperfect CSI. First, pilot tones were sent through a matrix generated by the outdated CSI at the transmitter and second, ZF or MMSE receiver is implemented at the receiver to eliminate the self-interference. For multiuser MIMO downlink systems with

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BD, [6] has investigated the effects of imperfect CSI on the capacity. The performance of multiuser MIMO systems strongly depends on the correlation between the estimated and the real CSI. Therefore the imperfect CSI affects the performance greatly.

In this paper, we propose a method to suppress the interferences among users and data streams caused by imperfect CSI at transmitter for a multiuser MIMO system with BD. First, the transmitter sends pilot signals through a linear spatial precoding matrix so as to make possible that the receiver can estimate the channel and the interference of other users. Therefore the receiver can exploit a linear prefilter to suppress the interference of other users as well as the self-interference of different data streams and improve the system performance. The remainder of this paper is organized as follows. In Section 2 the channel model is given. After that we analyze the effect of imperfect CSI on multiuser MIMO systems with BD in Section 3.1. Then the proposed method is presented in Section 3.2. Numerical examples are given in Section 4, while Section 5 contains our conclusions.

2 System Model

Consider a multiuser MIMO downlink system with BD, where a base station (BS) equipped with \( m \) transmit antennas transmits to \( K \) mobile users, each equipped with \( n_i \) \((i = 1, \ldots, K)\) antennas, on the assumption that \( m \geq \sum_{i=1}^{K} n_i \). The channels are assumed to be quasi-static\(^1\) flat fading and denoted by \( \mathbf{H}_k = \{h_{ij}(k)\}_{n_i \times m} \), where \( h_{ij}(k) \) is the channel gain from the \( i \)th transmit antenna to the \( j \)th receive antenna of the \( k \)th user. Let \( \mathbf{b}_k \) denote the \( n_k \times 1 \) transmit signal vector of user \( k \). This signal vector is first multiplied by a \( m \times n_k \) matrix \( \mathbf{R}_k \) and then transmitted through \( m \) transmit antennas. At the receiver of user \( k \), the received signal vector \( \mathbf{y}_k \) \( n_k \times 1 \) is processed by a linear filter \( \mathbf{R}_k \) to generate the output data vector \( \mathbf{b}_k \) as

\[
\tilde{\mathbf{b}}_k = \mathbf{R}_k \left( \mathbf{H}_k \sum_{i=1}^{K} \mathbf{T}_i \mathbf{b}_i + \mathbf{n}_k \right),
\]

where \( \mathbf{n}_k \) denotes the AWGN term which are modelled as i.i.d. zero mean complex Gaussian random variables with \( \sigma_n^2 \) variance, and \( \mathbf{b}_i \) satisfies \( \mathbb{E}[\mathbf{b}_i \mathbf{b}_i^H] = \mathbf{I}_{n_k} \).

For multiuser MIMO downlink systems with BD, the BS separates the users in spatial domain by selecting a set of precoding matrices \( \{\mathbf{T}_i\}_{i=1}^{K} \) which is generated by exploiting CSI at the transmitter under the constraint that users do not interfere with others, i.e. \( \mathbf{H}_i \mathbf{T}_i = 0 \forall j \neq i \) as in [11]. Define the \( \mathbf{H}_j \) as

\[
\mathbf{H}_j = [\mathbf{H}_1^T \cdots \mathbf{H}_{j-1}^T \mathbf{H}_{j+1}^T \cdots \mathbf{H}_K^T]^T,
\]

where superscript \( T \) is used to indicate transpose operation. The zero-interference constraint forces \( \mathbf{T}_j \) to lie in the null space of \( \mathbf{H}_j \). The singular value decomposition (SVD) of \( \mathbf{H}_j \) is

\[
\tilde{\mathbf{H}}_j = [\tilde{\mathbf{U}}_j^{(1)} \tilde{\mathbf{V}}_j^{(0)}] \left[ \begin{array}{cc} \hat{\Sigma}_j & 0 \\ 0 & 0 \end{array} \right] [\tilde{\mathbf{V}}_j^{(1)} \tilde{\mathbf{V}}_j^{(0)}]^H,
\]

where superscript \( H \) is used to indicate conjugate transpose operation. The right singular vectors \( \tilde{\mathbf{V}}_j^{(0)} \) form a basis for the null space of \( \tilde{\mathbf{H}}_j \). Therefore, \( \tilde{\mathbf{H}}_j \mathbf{V}_j^{(0)} = 0 \), i.e., \( \mathbf{H}_j \mathbf{V}_j^{(0)} = 0 \forall i \neq j \). The precoding matrix should be written as \( \mathbf{T}_j = \mathbf{V}_j^{(0)} \mathbf{A}_j \), where \( \mathbf{A}_j \) is used for further optimization. Denoted by \( \tilde{\mathbf{H}}_j = \mathbf{H}_j \mathbf{V}_j^{(0)} \), the equivalent channel matrix of user \( j \) has its SVD as

\[
\mathbf{H}_j = [\mathbf{U}_j^{(1)} \mathbf{U}_j^{(0)}] \left[ \begin{array}{cc} \hat{\Sigma}_j & 0 \\ 0 & 0 \end{array} \right] [\mathbf{V}_j^{(1)} \mathbf{V}_j^{(0)}]^H.
\]

The precoding matrix is then written as

\[
\mathbf{T}_j = \mathbf{V}_j^{(0)} \mathbf{A}_j = \mathbf{V}_j^{(0)} \mathbf{P}_j,
\]

where \( \mathbf{P}_j \) is a diagonal matrix defining power allocation for user \( j \) according to water-filling:

\[
[P_j]_{k,h} = \max \left( 0, \mu - \frac{\sigma_n^2}{|\mathbf{H}_j|^2} \right),
\]

where \( \mu \) is from the power constraint \( \sum_{j=1}^{K} \text{Trace}(\mathbf{P}_j) = P_t \), where \( P_t \) is the total transmit power.

The received signal of user \( k \) is then processed by a linear filter \( \mathbf{R}_k = \mathbf{U}_j^{(1)H} \) and becomes

\[
\tilde{\mathbf{b}}_k = \mathbf{U}_j^{(1)H} \left( \mathbf{H}_k \sum_{i=1}^{K} \mathbf{V}_i^{(0)} \mathbf{V}_j^{(1)} \mathbf{P}_i \mathbf{b}_i + \mathbf{n}_k \right) + \sum_{j \neq k} \mathbf{P}_j \mathbf{b}_j + \mathbf{U}_j^{(1)H} \mathbf{n}_k
\]

In many practical systems, water-filling power allocation puts a high demand on the linear range of transmit power amplifiers, which is extremely costly, especially for multiple antenna systems. Therefore we also consider the scheme with equal power allocation and have

\[
\tilde{\mathbf{b}}_k = \sqrt{\frac{P_t}{m}} \mathbf{R}_k \mathbf{H}_k \mathbf{V}_j^{(0)} \mathbf{b}_j + \mathbf{R}_k \mathbf{n}_k,
\]

where \( \mathbf{R}_k \) can be a linear MMSE receiver as

\[
\mathbf{R}_k = \left( \mathbf{V}_j^{(0)H} \mathbf{H}_k^H \mathbf{H}_j \mathbf{V}_j^{(0)} + \frac{\sigma_n^2 m}{P_t} \right)^{-1} \mathbf{V}_j^{(0)H} \mathbf{H}_k^H.
\]
3 TDD Multiuser MIMO System with BD over Time-varying channels

3.1 Impact of Outdated CSI on MIMO downlink

In TDD systems, the reciprocity property of the channel is used to obtain CSI at the transmitter. Assume that the users transmit data to BS at time slot \( t \). Pilot symbols are transmitted by different users prior to data transmission through channel \( \mathbf{H}_k(t) \) (\( 1 \leq k \leq K \)). The BS then estimates the CSI by the pilot symbols. Although the channel estimation from pilot symbols is not perfect in practice and results in channel estimation errors, the impact of the errors on system performance is not severe, especially for the system with sufficient long pilot symbols [6]. Thus we assume the estimation at this stage is perfect for simplicity. In the next time slot \( t + 1 \), the BS uses the CSI estimated at time slot \( t \) for downlink transmission. The delay between the estimation of the channels and the transmission is the length of a time slot \( T_s \). When the channel is time-varying, the CSI at BS is outdated and thereby leading to

\[
\tilde{b}_k(t+1) = \mathbf{U}^{(1)}H_k(t+1) \left( \mathbf{H}_k(t+1) \sum_{i=1}^{K} \mathbf{V}_i^{(0)}(t) \mathbf{V}_i^{(1)}(t) \mathbf{P}_i(t) \mathbf{b}_i(t+1) \right) + \mathbf{n}_k(t+1),
\]

where \( \mathbf{b}_k(t) \) is the transmitted symbol of user \( k \) at time slot \( t \), and \( \mathbf{H}_k(t) \) is the channel estimated at time slot \( t \). With the channel estimated at time slot \( t \), we obtain the received signal vector of user \( k \) as

\[
\mathbf{y}_k(t+1) = \mathbf{H}_k(t+1) \mathbf{b}_k(t+1) + \mathbf{n}_k(t+1).
\]

The first term in (13) includes inter-stream interference of user \( k \), and the second term is the inter-user interference. From Jake’s model [9], the correlation between the channel gains in two slots is \( \rho_j = 2\pi F_d T_s \), where \( \rho_j \) is the zeroth-order Bessel function of the first kind and \( F_d \) is the Doppler frequency. When the channels are varying fast or the time delay \( T_s \) is large, the correlation is not high enough that the channel estimation errors are large and degrade the performance greatly.

3.2 Proposed Method

From the last section, we have shown that the outdated CSI at BS results in inter-stream and inter-user interference which greatly affect the system performance. In order to reduce the degradation of the interferences, we propose a pilots architecture that is shown in Fig. 1. When the BS is transmitting data to users at time slot \( t + 1 \), the pilot symbols are first processed by the precoding matrices \( \{ \mathbf{T}_i(t) \}_{i=1}^{K} \), which are generated by the BS from the outdated CSI. For user \( j \), the channel is estimated as

\[
\mathbf{H}_j(t+1) = \mathbf{H}_j(t+1) \mathbf{T}_i(t).
\]

The equivalent matrix from user \( j \) to user \( j \) as

\[
\mathbf{y}_j(t+1) = \sum_{i=1}^{K} \mathbf{H}_{ij}(t+1) \mathbf{b}_i(t+1) + \mathbf{n}_j(t+1).
\]

By the proposed pilots architecture, each user can know the interferences caused by the outdated CSI at the BS exactly and thereby exploit MUD receiver to reduce the interferences. The performance can be further improved by using nonlinear detection method, e.g., successive interference cancellation. For simplicity, we only consider linear method in this paper. We can obtain the linear filter of user \( j \), \( \mathbf{R}_j(t+1) \) from Minimum mean-square error (MMSE) criteria as

\[
\min \text{Trace}(E[(\mathbf{R}_j(t+1) \mathbf{y}_j(t+1) - \mathbf{b}_j(t+1))])
\]

subject to: \( \text{Trace}([\mathbf{R}_j(t+1) \mathbf{R}_j(t+1)^H]) = 1 \). (17)

Then we have

\[
\mathbf{R}_j = \mathbf{H}^H_{jj} \sum_{i=1}^{K} \mathbf{H}_{ij} \mathbf{H}^H_{ij} + \sigma^2_n I_n, \quad \text{subject to: } \text{Trace}([\mathbf{R}_j(t+1) \mathbf{R}_j(t+1)^H]) = 1.
\]
Note that using $R_T$ at the receiver can only suppress a part of interference, since the number of interference sources is normally larger than the number of receive antennas in this case. In [1], the authors conclude that when the inter-user interference is large, it is capacity optimal to only transmit one data stream for each user. Thus when $F_dT_s$ is large, it is optimal to use one stream transmission.

4 Numerical Examples

In the simulations, we consider a MIMO downlink system with a BS equipped with 6 antennas transmitting data to 3 users each equipped with 2 antennas. Jake’s model is used to simulate the time-varying channels. We also assume independent fading among all the antennas. For comparison purpose, we simulate the following reference systems:

1. Open loop system: The BS exploits no CSI for transmission and only transmit data to one user at a time.
2. Multiuser MIMO with perfect CSI: The BS can estimate the CSI perfectly and use BD method to transmit to 3 users simultaneously.
3. Multiuser MIMO with outdated CSI: The BS only has outdated CSI and still uses BD method for transmission.
4. One stream scheme [1]: In this system, only one stream is used for each user all the time.

Monte Carlo simulations are used to compare the sum capacity of different reference systems. We generate large number of channel realizations and compute the SINR of each substream of each user. The interferences are assumed to be Gaussian, thereby leading the capacity of each substream can be computed by Shannon formula. Assuming a typical time-duplex delay $T_s = 1$ ms, we have different $F_dT_s$ in scenarios with different $F_d$.

![Figure 3. Sum capacity of equal power system with $F_dT_s = 0.02$](image)

In Fig. 2, we plot the sum capacity of water-filling system with $F_dT_s = 0.02$ against SNR. The outdated CSI at BS greatly degrades the performance of multiuser MIMO downlink, especially for high SNR. For 25 dB cases, the capacity gap between the system with perfect and imperfect CSI is about 6 bps/Hz. The proposed method can obviously improve the performance and the improvement increases as the SNR increases. Fig. 2 plots the sum capacity of equal power system with $F_dT_s = 0.02$. The capacity of equal power system is less than that of the water-filling system, especially for low SNR. The performance enhance achieved by the proposed method is larger in equal power systems. In both these two systems, one stream scheme is not better than other multiuser schemes, since the inter-user interference caused by outdated CSI is not that severe.

In Fig. 4, we plot the sum capacity of water-filling system with $F_dT_s = 0.04$. The capacity degrades more in comparison to Fig. 2. The proposed method achieves more performance improvement as the $F_dT_s$ increases.

Fig. 5 plots the sum capacity of water-filling system with fixed SNR 15 dB. When $F_dT_s$ is more than 0.08, the open loop transmission outperforms the multiuser systems with imperfect CSI. The proposed scheme can enlarge the application range of multiuser MIMO systems. When $F_dT_s$ is less than 0.12, using multiuser system is still more efficient that the open loop scheme. Note that one stream scheme achieves better performance than the proposed scheme for
large $F_d T_s$, but its performance is worse than the open loop scheme in such cases, therefore is still not suitable for practical application in such cases.

![Figure 4. Sum capacity of water-filling system with $F_d T_s = 0.04$](image)

![Figure 5. Sum capacity of water-filling system with fixed SNR 15 dB](image)

### 5 Conclusion

In this paper we have investigated the effect of outdated CSI at the transmitter on multiuser MIMO systems with BD. A pilots architecture and linear MUD-MMSE receiver are proposed jointly to suppress the inter-stream and inter-user interferences at the receiver side. The simulation results have evaluated the efficiency of the proposed method. When the $F_d T_s$ of the system is in a particular range, the proposed method can achieve better performance in comparison with other schemes.

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### References