Random Beamforming with Multi-beam Selection for MIMO Broadcast Channels

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Abstract—Previous work has shown that the capacity region of the Gaussian MIMO broadcast channels is achieved by dirty paper coding (DPC). However, due to high computation complexity of DPC and infeasibility of perfect channel state information (CSI) at the transmitter in many applications, this paper focuses on a reduced complexity transmission scheme named orthonormal random beamforming (ORBF) [16], which only requires partial CSI feedback at the transmitter. Different from the previous work, we analyze the performance of ORBF with moderate number of users and total transmit power constraint. The analysis results show that ORBF scheme is efficient under low SNR. Then we propose a multi-beam selection (MBS) scheme, which selects only the best subset of all the beams to maximize the sum-rate capacity under low SNR. The simulation results show that the proposed MBS scheme achieves great performance improvement when the SNR is low and the number of users is not very large.

Index Terms—MIMO, Broadcast Channels, Random beamforming, Multi-beam selection

I. INTRODUCTION

Growing demand of high speed wireless Internet access has led to extensive research on multiple input multiple output (MIMO) systems. Previous work [1] [2] has indicated remarkable spectral efficiency of single-user MIMO links. However, there is recent interest in the role of MIMO systems in a multiuser network environment, especially in broadcast scenarios [3]. Recent results in [4] [5] have shown that the sum-rate capacity is achieved by dirty paper coding (DPC) [6]. Moreover, it has been shown that DPC in fact achieves the capacity region of the Gaussian MIMO broadcast channel [7].

However, the DPC schemes, especially in the multiuser context, are difficult to implement in practical systems due to high computational burden of successive encoding and decoding. A reduced-complexity scheme referred as zero-forcing dirty paper coding (ZFDPC) is proposed in [5] [8]. It uses QR decomposition combined with DPC at the transmitter to optimize the sum-rate capacity and provides performance very close to the Sato bound [9]. A linear suboptimal strategy that can serve multiple users at a time like DPC, but with much reduced complexity, is zero-forcing beamforming (ZFBF) scheme [10]. If multiple antennas are used at the receiver, block diagonalization (BD) [11] is a more efficient method to achieve sum-rate capacity.

While the above work achieves sum-rate capacity, it assumes perfectly known channel state information (CSI) at the transmitter. In many applications, the assumption is not reasonable. This is especially true, if the number of transmit antennas and the number of users are large, or the channels vary rapidly. On the other hand, using multiple transmit antennas yields no gain if no CSI is available at the transmitter. Therefore, in this paper we focus on the transmission schemes that require only partial CSI at the transmitter.

In [12] [13], stream scheduling is proposed for multiuser MIMO spatial multiplexing systems with partial feedback. But their work assumes that the number of receive antennas is no less than the number of transmit antennas, which is not reasonable for many downlink systems, e.g., cellular systems. Another method named random beamforming is proposed in [14] to attain coherent beamforming capacity while requiring only signal-to-noise-plus-interference ratio (SINR) as the feedback. This method uses a random beamformer and allocates resources to one user who has the best channel condition. In [15], the authors extend the random beamforming method to MIMO systems. However, in their work, only one user is transmitted at a time. An orthonormal random beamforming (ORBF) scheme with multiple random beams for MIMO broadcast channels is then presented in [16]. In ORBF, each user reports to the transmitter its maximum SINR along with the index number of the beam in which the SINR is maximized. Based on asymptotic analysis, [16] shows that, ORBF achieves the optimal sum-rate capacity of DPC for a large number of users. But for a moderate number of users, i.e., no more than 64, ORBF only achieves slight performance gain [17]. Therefore, [18] proposes an improved random beamforming scheme called multi-user diversity and multiplexing (MUDAM). However, MUDAM causes long feedback delay since the users need to feed back CSI many times before all the beamforming vectors are determined. In the scenarios with rapidly moving users, the channel states may change before all the beamforming vectors are generated and thus MUDAM is not suitable for such cases.

Therefore, this paper considers the ORBF scheme with moderate number of users and low feedback delay requirement. Besides the assumption on moderate number of users, we also assume the total transmit power constraint, which is different from [16]. Since the ORBF scheme only has partial CSI at the transmitter, it can not perfectly cancel the interference between the simultaneously transmitting users. As a result, when the average SNR increases, i.e., the systems tend to be interference-limited, the performance of ORBF may degrade. That is to say, ORBF scheme is more efficient under low average SNR. Moreover, when average SNR is low, power allocation becomes important and may improve the system performance greatly.
Therefore, we propose a multi-beam selection (MBS) scheme for the random beamforming systems. In MBS, the transmitter selects the number and the subset of the multiple beams, and allocates power to the selected beams to maximize the sum-rate capacity. Note that the proposed MBS scheme does not require extra feedback information compared to the ORBF systems.

The paper is organized as follows. In Section II the channel model is given. After that we analyze the performance of ORBF scheme with moderate number of users in Section III. Then the multi-beam selection scheme is proposed in Section IV. Numerical examples are given in Section V, while Section VI contains our conclusions.

II. SYSTEM MODEL

We consider a MIMO Gaussian broadcast channel with \( K \) receivers equipped with \( m_r \) antennas and a transmitter with \( m_t \) antennas. For a typical cellular system, the number of users is larger than the number of transmit antennas and also the number of antennas in the base station is greater than the number of antennas in the receiver. Thus we assume \( K > m_t > m_r \). The channels are assumed to be quasi-static\(^1\) flat fading and denoted by \( \mathbf{H}_k = \{ h_{ij}(k) \}_{m_r \times m_t} \), where \( h_{ij}(k) \) is the channel gain from the \( j \)th transmit antenna to the \( i \)th receive antenna of the \( k \)th user and is assumed to be i.i.d. zero mean complex Gaussian random variable with unitary variance. The transmit generating \( m_b \) (\( 1 \leq m_b \leq m_t \)) random orthonormal vectors \( \phi_m \ (m_t \times 1) \) according to an isotropic distribution \[19\]. Given the \( n \)th transmitted symbol \( s_n \), the received signal of the \( k \)th user is

\[
y_k = \sum_{m=1}^{m_b} \mathbf{H}_k \phi_m s_n + \mathbf{n},
\]

where \( \mathbf{n} \) denotes the AWGN term which are modelled as i.i.d. zero mean complex Gaussian random variables with unitary variance. The long-term average SNR of the \( k \)th user is denoted by \( \rho_0 \) and \( \mathbb{E}[s^h s] = \rho_0 \).\(^2\) We assume equal power allocation to each beam, so that \( \mathbb{E}[s^h_n s_n] = \frac{\rho_0}{m_b} (1 \leq m \leq m_b) \).

III. ANALYSIS OF ORTHONORMAL RANDOM BEAMFORMING (ORBF) SCHEME

In this section, we analyze the sum-rate capacity of the ORBF scheme with fixed \( K \) and \( m_t \). For simplicity, we first assume \( m_r = 1 \) in the derivation and then extend the conclusion to the cases with \( m_r > 1 \).

Perfect CSI is assumed to be available at the receiver. Therefore, the \( k \)th receiver can compute the SINR of the \( n \)th beam by assuming that \( s_n \) is the desired signal and the others are interference as

\[
\text{SINR}_{kn} = \frac{|| \mathbf{H}_k \phi_m ||^2}{m_b/\rho_0 + \sum_{i \neq m} || \mathbf{H}_k \phi_i ||^2}.
\]

Each receiver feeds back its maximum SINR with the index in which the SINR is maximized. The transmitter then assigns \( s_m (1 \leq m \leq m_b) \) to the user with the best corresponding SINR. Therefore, the sum-rate capacity of ORBF scheme with \( m_b \) beams is

\[
R(m_b) \approx \mathbb{E} \left[ \sum_{i=1}^{m_b} \log \left( 1 + \max_{1 \leq k \leq K} \text{SINR}_{ki} \right) \right] = m_b \mathbb{E} \left[ \log \left( 1 + \max_{1 \leq k \leq K} \text{SINR}_{ki} \right) \right],
\]

where the approximation is from the fact that we neglect the small probability that users may be the strongest user for more than one signal \( s_m \).

Since \( \phi_m \) are orthonormal vectors, \( \mathbf{H}_k \phi_m \) is i.i.d. zero mean complex Gaussian random variables with unitary variance over \( k \) and \( m \). Thus,

\[
\text{SINR}_{km} = \frac{|| \mathbf{H}_k \phi_m ||^2}{m_b/\rho_0 + \sum_{i \neq m} || \mathbf{H}_k \phi_i ||^2} = \frac{z}{m_b/\rho_0 + y},
\]

where \( z \) and \( y \) have \( \chi^2(2) \) and \( \chi^2(2m_b - 2) \) distribution respectively. The probability distribution function (pdf) of SINR \( \text{SINR}_{km} \) is

\[
f_s(x) = \int_0^\infty f_{\text{SINR}_{km}}(x|y) f_Y(y) dy = \frac{e^{-m_b x}}{(1 + x)^{m_b}} \left( m_b - 1 + \frac{m_b}{\rho_0} (1 + x) \right).
\]

Therefore, the cumulative distribution function (cdf) of SINR \( \text{SINR}_{km} \) is

\[
F_s(x) = 1 - \frac{e^{-m_b x}}{(1 + x)^{m_b}} \left( m_b - 1 + \frac{m_b}{\rho_0} (1 + x) \right),
\]

Since SINR \( \text{SINR}_{km} \) are i.i.d. random variables over \( k \), the cdf of \( \max_{1 \leq k \leq K} \text{SINR}_{km} \) is \( (F_s(x))^K \). Hence, the sum-rate capacity with \( m_b \) beams is

\[
R(m_b) = m_b \int_0^\infty \log(1 + x) dF_s^K(x).
\]

Furthermore, the sum-rate capacity of the systems is

\[
R = \max_{1 \leq m_b \leq m_t} R(m_b) \approx \max_{1 \leq m_b \leq m_t} m_b \int_0^\infty \log(1 + x) dF_s^K(x).
\]

We now begin to evaluate the sum-rate capacity and find out the optimal number of \( m_b \).

**Proposition 1:** For fixed \( K \) in low SNR cases, i.e., \( \rho_0 \to 0 \), we have

\[
m_b = \arg \max_{1 \leq m_b \leq m_t} R(m_b) = m_t.
\]

**Proof:** When \( \rho_0 \to 0 \), \( \text{SINR}_{km} \approx || \mathbf{H}_k \phi_m ||^2 \rho_0 \). Resulting the pdf and cdf of SINR \( \text{SINR}_{km} \) as

\[
f_s(x) = \frac{m_b}{\rho_0} e^{-m_b x} \quad \text{and} \quad F_s(x) = 1 - e^{-m_b x} \rho_0^x/m_b,
\]

\(^1\)Quasi-static means that the channel are constant over a frame length and changed independently between different frames.

\(^2\)We use \((\cdot)^t\) and \((\cdot)^*\) to denote the transpose and conjugate transpose operation respectively, \((\cdot)^*\) to denote the conjugate of a number, and \(\mathbb{E}[]\) to denote the expectation operator.
respectively. Letting \( u = F_s^K(x) \), we have
\[
x = \frac{\rho_0}{m_b} \log \left( 1 - u^{1/K} \right).
\] (12)

Substituting (10), (11), and (12) in (7), we get
\[
R(m_b) = m_b \int_0^1 \log \left( 1 + \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} \right) du.
\] (13)

Therefore the differential of \( R(m_b) \) over \( m_b \) is
\[
\frac{\partial R(m_b)}{\partial m_b} = \int_0^1 \log \left( 1 + \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} \right) du - \int_0^1 \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} du
\]
\[
> \int_0^1 \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} du - \int_0^1 \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} du
\]
\[
= \int_0^1 \left( \frac{\rho_0}{m_b} \log \frac{1}{1 - u^{1/K}} \right)^2 du > 0.
\] (14)

As a result, \( R(m_b) \) increases as \( m_b \) increases, leading \( \hat{m}_b = m_1 \).

Although the ORBF scheme can not perfectly cancel the interference between the simultaneously transmitting users, in the cases with low SNR, the interference is small enough. Thus transmitting to more users simultaneously does not generate severe interference. Therefore the sum-rate capacity is maximized by using \( m_t \) beams.

**Proposition 2:** For fixed \( K \) in high SNR cases, i.e., \( \rho_0 \to \infty \), we have
\[
\hat{m}_b = \arg \max_{1 \leq m_b \leq m_t} R(m_b) = 1.
\] (15)

**Proof:** When \( \rho_0 \to \infty \), we have
\[
\text{SINR}_{km} \approx \frac{|H_k \phi_m|^2}{\sum_{i \neq m} |H_k \phi_i|^2}, \quad m_b > 1,
\] (16)

whose pdf and cdf are
\[
f_s(x) = \frac{m_b - 1}{(1 + x)^{m_b}},
\] (17)
\[
F_s(x) = 1 - \frac{1}{(1 + x)^{m_b-1}},
\] (18)
respectively. Letting \( u = F_s^K(x) \), we have
\[
x = \left( \frac{1}{1 - u^{1/K}} \right)^{m_b-1} - 1.
\] (19)

Substituting (17), (18) and (19) in (7), we obtain
\[
R(m_b) = \frac{m_b}{m_b-1} \int_0^1 \log \frac{1}{1 - u^{1/K}} du, \quad m_b > 1
\] (20)

which is obviously monotonic decreasing over \( m_b \). Thus \( R(2) = \max_{2 \leq m_b \leq m_t} R(m_b) \). When \( m_b = 1 \), we have
\[
R(1) = \int_0^\infty \log(1 + x) \frac{1}{\rho_0} e^{-\rho_0 x} dx.
\] (21)

For sufficient large \( \rho_0 \), we have
\[
R(1) > R(2) = \int_0^1 \log \frac{1}{1 - u^{1/K}} du.
\] (22)

Therefore \( \hat{m}_b = 1 \) as \( \rho_0 \to \infty \).

Remark that in the cases with high SNR, (20) shows that the sum-rate capacity of the system with \( m_b \geq 2 \) does not affect by \( \rho_0 \), i.e., the system is interference-limited. The ORBF scheme with multiple beams is not efficient in such cases. Thus it is better to use single beam at the transmitter.

For the cases with \( m_r > 1 \), we treat each receive antenna as an independent user. By this way, we effectively have \( m_t \cdot K \) users with single antenna. Therefore, for these cases, the formulation of the problem is the same as that in the above discussion with only difference being that \( K \) is replaced by \( m_t \cdot K \).

**IV. MULTI-BEAM SELECTION SCHEME**

From Proposition 1 and 2 in last section, ORBF scheme is more efficient in the low SNR cases. Moreover, when the SNR is low, power control becomes important and can improve the system performance greatly. Thus we propose multi-beam selection (MBS) for the ORBF scheme. Different from ORBF scheme with variable number of random beams, in MBS scheme the transmitter always generates \( m_t \) orthonomal random beams and only the best \( m_b \) beams of all the beams are used to improve the power efficiency. The receivers estimates the SINR for the \( m_t \) random beams as
\[
\text{SINR}_{km} = \frac{|H_k \phi_m|^2}{m_b/\rho_0 + \sum_{i \neq m} |H_k \phi_i|^2}, \quad 1 \leq m \leq m_t.
\] (23)

Denoting the maximized estimated SINR of the \( k \)th user as
\[
\text{SINR}_{km} = \frac{|H_k \phi_m|^2}{m_b/\rho_0 + \sum_{i \neq m, i \in B} |H_k \phi_i|^2}, \quad k \in U,
\] (25)

we get the descending ordered \( \text{SINR}_{km} \) as \( \text{SINR}_{km}(l), (1 \leq l \leq K) \). Then the \( m_b \) users with highest \( \text{SINR}_{km}(l) \) are selected for transmitting. Denote the selected beams and the users subset as \( B \) and \( U \) respectively, we get the SINR of the selected users as
\[
\text{SINR}_{km} = \frac{|H_k \phi_m|^2}{m_b/\rho_0 + \sum_{i \neq m, i \in B} |H_k \phi_i|^2}, \quad k \in U,
\] (25)

which is different from the estimated SINR. However, in the low SNR cases, the difference between the estimated SINR and the SINR becomes small. The \( \text{SINR}_{km} \) is approximately \( |H_k \phi_m|^2 \rho_0/m_b \), and the sum-rate capacity of the MBS scheme with \( m_b \) selected beams is
\[
R_s(m_b) \approx \mathbb{E} \left[ \sum_{l=1}^{m_b} \log(1 + \text{SINR}_{km}(l)) \right].
\] (26)
Fig. 1 plots the sum-rate capacity of ORBF scheme with different number of random beams for maximized sum-rate capacity. From which we can also obtain the optimal number of selected beams for maximizing sum-rate capacity.

The cdf of $\text{SNR}_{k}(l)$ is [20]

$$F_l(x) = \sum_{i=K-l+1}^{K} \binom{K}{K-i+1} (F_{s}^{m_{l}}(x))^{(K-i+1)} (1 - F_{s}^{m_{l}}(x))^{(i-1)}, \quad (27)$$

and the sum-rate capacity is

$$R_s = \max_{1 \leq m_{l} \leq m} \sum_{l=1}^{m_{b}} \int_{0}^{\infty} \log(1 + x) dF_l(x), \quad (28)$$

from which we can also obtain the optimal number of selected beams for maximized sum-rate capacity.

V. NUMERICAL EXAMPLES

This section verifies our analysis results by Monte Carlo simulations. For simplicity, we assume $m_{l} = 1$ in the simulations. Fig. 1 plots the sum-rate capacity of ORBF scheme with different number of random beams. Fig. 1(a) shows the sum-rate capacity under low SNR. The results show that, the capacity increases as the number of random beams increases with $K = 60$, which matches Proposition 1 well. However, when $K = 20$, the optimal number of beams is 5 and then the capacity decreases as the number of beams increases to more that 5. The reason is that the number of users is not sufficient resulting in high probability that users may be the strongest user for more than one signal $s_{m}$. Thus the approximation in (3) is not reasonable. In this case, due to lack of multiuser diversity gain, using more beams can allocate the power to the beams with low gain, and therefore decreases the sum-rate capacity. Fig. 1(b) shows the sum-rate capacity under high SNR. The results also match the analysis of Proposition 2 well. For $m_{b} \geq 2$, the capacity decreases as $m_{b}$ increases. For the case with $K = 60$, $\rho_{0}$ is not sufficient large to make the capacity of single antenna system larger than that of the multi-beam system. But for $K = 20$, the capacity is maximized by using single antenna at the transmitter, i.e., ORBF scheme is not suitable for the cases with high SNR.

Fig. 2 plots the sum-rate capacity of the ORBF and MBS schemes with $m_{l} = 6$. We also plot the performance gain of MBS in comparison with ORBF scheme in fig. 3. The simulation results show that the proposed MBS scheme achieves obviously performance enhancement, especially for the low SNR cases with moderate number of users. For the case with $-8dB$ SNR and 10 users, the proposed MBS scheme has 40% performance gain than the ORBF scheme. The performance gain decreases as the SNR increases. When the SNR is $12dB$, the MBS scheme achieves no gain for the system with 10 users. Furthermore, the performance gain also decreases as the number of users increases. However, a wireless system with channel coding like Turbo or LDPC always works in low SNR. Therefore, our proposed scheme is a good choice for such cases with moderate users.

VI. CONCLUSION

In this paper, we have analyzed the sum-rate capacity of a MIMO broadcast system with ORBF scheme and moderate
number of users. The analysis results have shown that it is not efficient to use multiple simultaneously transmitting beams under high SNR. For the cases with low SNR, we have proposed a multi-beam selection scheme to improve the sum-rate capacity. The proposed MBS scheme selects only the best $m_b$ beams of all the $m_t$ beams based on the feedback information. From the simulation results, we have shown that MBS scheme achieves great performance improvement when the SNR is low and the number of users is in moderate level.

REFERENCES