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Abstract—This paper studies the resource allocation problem of a single cell powered jointly by renewable energy and power grid over a given time period (e.g., 24 hours), using statistical information of traffic intensity and harvested energy. Specifically, the problem is formulated as minimizing the average grid power input while satisfying users’ quality of service (outage probability) requirements. We analyze the outage probability, and solve the grid power minimization problem indirectly by obtaining a power-outage tradeoff curve using the dynamic programming (DP) approach. Some heuristic algorithms are proposed and compared with the DP algorithm by simulations. The results show that the DP algorithm greatly reduces the grid power consumption compared with the heuristic methods, among which the joint traffic-energy-aware resource allocation performs closest to the optimal solution.

I. INTRODUCTION

Energy harvesting technology for wireless data transmission, which exploits renewable energy (e.g., solar energy, wind energy, etc.) from the surrounding environment, is one of the candidate solutions to improve the energy efficiency of communication systems [1]. Due to the limited availability of harvested energy as well as uncertainty about timing and variability in the quantity of energy collected, there is a tradeoff between the quality of service (QoS) and the available power budget. On the one hand, increasing active wireless resources enhances the system capacity, but on the other hand increases the probability of energy depletion, which will ultimately degrade users’ QoS. Hence, in energy harvesting systems, wireless resource allocation should be re-examined and optimized jointly considering the traffic profile, the users’ QoS requirement, and the renewable energy statistics.

In the literature, resource allocation with energy harvesting has attracted extensive study recently. The offline optimal power allocation policy in a non-fading channel is analyzed in [2]. The work is extended to the fading channel scenario in [3], where the structure of the optimal power allocation is interpreted as the directional water-filling policy. However, it is difficult to know the energy arrival profile in advance due to source’s uncertainty. Consequently, resource allocation using statistics of the harvested energy is more practical. Ref. [4] considers a cross-layer resource allocation problem to maximize the total system utility using a Markov decision process (MDP) approach [5]. The packet dropping and blocking probabilities are analyzed with different sleep and wake-up strategies using queuing theory in sensor/mesh networks with solar power [6]. In [7], it is shown that the wireless link performance is strongly influenced by the renewable energy profile, and parameter adaptation is considered to improve the performance. The maximum stable throughput is studied and derived in closed-form in cognitive radio networks [8] and cooperative networks [9], respectively. Nevertheless, most of existing work focuses on link level analysis, while the problem of how to efficiently utilize the harvested energy from a network point of view still remains open.

With a stable power supply (grid power), the influence of traffic intensity on the network performance is analyzed in [10]. In this paper, we take the energy uncertainty issue into consideration, and study the wireless resource allocation problem in cellular networks according to the statistics of network traffic and energy profile. A mixed power supply from both renewable energy and power grid is adopted. The grid power is used to guarantee users’ QoS service when the renewable energy is insufficient or unstable. The main contribution of this work is that for the first time, the relation between energy consumption and outage probability with long-term average information of traffic and renewable energy is studied.

Specifically, we analyze the outage probability of a single cell downlink with given average power budget. Based on the analysis, we formulate and solve the problem of average grid power input minimization with the users’ QoS (weighted outage probability) constraint for a pre-defined time period (e.g., 24 hours), using knowledge of the traffic load profile and the energy harvesting statistics. We solve the related unconstrained problem of minimizing a weighted combination of power consumption and outage probability by the dynamic programming (DP) [5] approach, and find the power-outage tradeoff curve. Hence, the minimum average grid power consumption and the optimal resource allocation policy for a target outage probability can be obtained depending on the curve. Then heuristic solutions are proposed and evaluated by numerical simulations.

The rest of the paper is organized as follows. Section II introduces the single-cell system model. The outage probability is defined and analyzed in Section III. In Section IV, we study
the average grid power minimization problem with a weighted outage probability constraint. Numerical results are presented in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

We consider a single cell with a radius of $R$, where the BS is powered jointly by an energy harvesting device and the power grid. The whole time line (e.g., 24 hours) is divided into $T$ slots. In slot $t$, the average harvested power is denoted as $P_H,t$, and the grid power is $P_G,t$. Assume the harvested energy is stored in an infinite capacity battery. The assumption is reasonable as the harvested energy is generally not sufficient for reliable network operation. Hence, in a real system, even though the battery capacity is finite, there is very low probability of battery overflow. The BS energy consumption in active mode is modeled as a constant power term plus a radio frequency (RF) related power, which is

$$P_{BS,t} = P_0 + \Delta P P_{RF,t},$$

where $P_0$ is the constant power including the baseband processor, the converter, the cooling system, etc., $\Delta P$ is the slope of the load dependent power consumption, and $P_{RF,t}$ is the total RF transmit power.

Assume the total wireless bandwidth $W_0$ is divided into $N$ orthogonal subcarriers, or in other words, resource blocks. The system will decide how many resource blocks are active. The resource blocks can also be divided in time domain, i.e., frames. The time domain resource allocation, where some frames are active for transmission while the others are in sleep mode [11], is left for future work. The RF power is a linear function of the number of active resource blocks $n_t$, i.e.,

$$P_{RF,t} = \frac{n_t}{N} P_T,t, \quad n_t \leq N,$$

where $P_T,t$ is the transmit power. In this paper, we assume the transmit power is fixed at $P_T,t = P_T$ to keep the cell coverage. In a multi-cell scenario, $P_T,t$ can be adjusted to enable cell zooming [12]. Substituting $P_{RF,t}$ in Eq. (1) with (2), we get the resulting resource allocation power model

$$P_{BS,t} = P_0 + \frac{n_t}{N} \Delta P P_T.$$

The user’s QoS is measured by the outage probability, defined as the probability that the user throughput requirement is not satisfied, which is derived as:

$$\bar{p}_{out,t} = E_{m,h,l} \left[ \Pr \left( \frac{n_t W_0}{m N} \log \left( 1 + \frac{P_T |h|^2 \gamma^{-\alpha}}{\sigma^2} \right) < r_0 \right) \right]$$

where $h$ is Rayleigh fading random variable, $l$ is the distance between the BS and the user, $\gamma$ is the pathloss constant and $\alpha$ is the pathloss exponent, and $\sigma^2$ is the noise power.

III. OUTAGE PROBABILITY ANALYSIS

In this section, we analyze the outage probability given the statistics of data traffic and average power input. Recall that the outage probability in slot $t$ is the probability that the user throughput requirement is not satisfied, which is derived as:

$$\bar{p}_{out,t} = E_{m,h,l} \left[ \Pr \left( \frac{n_t W_0}{m N} \log \left( 1 + \frac{P_T |h|^2 \gamma^{-\alpha}}{\sigma^2} \right) < r_0 \right) \right]$$

where $P_T$ denotes the amount of arrived energy. The expectation in (5) is taken over the number of active users and their positions, channel fading, as well as energy statistics. Eq. (6) means that outage happens when either the target data rate is not achieved or the required power is not available. In (7), $\bar{p}_{out,t}$ is the conditional outage probability given that the required power consumption is satisfied, which is derived as

$$\bar{p}_{out,t} = E_{m,h,l} \left[ \Pr \left( \frac{n_t W_0}{m N} \log \left( 1 + \frac{P_T |h|^2 \gamma^{-\alpha}}{\sigma^2} \right) < r_0 \right) \right]$$

where $\bar{r}_0 = (N r_0)/W_0$ is the required spectrum efficiency with a single resource block. The scalar $p_{E,t}$ is the probability that the transmit power $P_T$ is available. If the power is not available, the BS will be turned into sleep mode in some frames with power $P_S$. We ignore the mode switching power among frames as it is not significant compared to the active mode power consumption. Then, we have

$$(1 - p_{E,t}) P_S + p_{E,t} P_{BS,t} \leq P_{in,t},$$

where $P_{in,t}$ is the average power (harvested energy, power grid, or mixed). It does not depend on the power distribution. If $P_{in,t} < P_{BS,t}$, (9) is satisfied with equality. Otherwise, the
power is always available, and \( p_{E,t} = 1 \). Hence, we get
\[
p_{E,t} = \min \left\{ \frac{P_{\text{in},t} - P_S}{P_{\text{BS},t} - P_S}, 1 \right\}. \tag{10}\]

Note that there is no closed-form expression of the outage probability as (8) contains the summation of an infinite sequence. However, it can be approximated by truncating into a sum of a finite sequence of terms instead since
\[
\int_0^\infty \frac{2^\gamma}{\nu^\gamma} \exp \left( -\frac{\eta}{\nu} \right) d\nu < 1, \text{ and for any } \delta > 0, \text{ there exists an } m_0, \text{ such that for any } m > m_0, \frac{e^{-\lambda m}}{m!} > \frac{e^{-\lambda (m+1)}}{(m+1)!} \to 0 \text{ and } \sum_{k=m}^{\infty} \frac{e^{-\lambda k}}{k!} < \delta.
\]

IV. POWER GRID ENERGY MINIMIZATION

A. Problem Formulation

We then consider the problem with time-variable traffic and energy arrival. The traffic intensity \( \lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_T\} \) and the energy harvesting rate \( P_H = \{P_{H,1}, P_{H,2}, \ldots, P_{H,T}\} \) are assumed to be constant in each slot \( t \), but can vary between slots. By adjusting the wireless resource allocation policies \( n = \{n_1, n_2, \ldots, n_T\} \) and the grid power inputs \( P_G = \{P_{G,1}, P_{G,2}, \ldots, P_{G,T}\} \), we can adapt the power usage in all the slots. The harvested energy can be reserved in an energy battery. Denote the energy in the battery at the beginning of slot \( t \) by \( E_{B,t} \), which is updated as
\[
E_{B,t+1} = \max \left\{ E_{B,t} + L_t (P_{H,t} + P_{G,t}) - p_{E,t} L_t P_{\text{BS},t} (1 - p_{E,t}) L_t P_S, 0 \right\}, \tag{11}\]
where \( L_t \) is the length of slot \( t \). Notice that we assume the grid energy is not stored in the energy buffer. Hence, due to the BS power consumption limitation, we have \( 0 \leq P_{G,t} \leq \max\{0, P_0 + \Delta P T - P_{H,t} - E_{B,t}/L_t\} \), i.e., the power grid is not plugged in if the harvested energy is enough.

The power supply is from instantaneous energy harvesting, the energy battery, or the power grid. Hence, the average power available in slot \( t \) is \( P_{\text{in},t} = P_{H,t} + P_{G,t} + E_{B,t}/L_t \), and we have
\[
p_{E,t} = \min \left\{ \frac{P_{H,t} + P_{G,t} + E_{B,t}/L_t - P_S}{P_{\text{BS},t} - P_S}, 1 \right\}. \tag{12}\]

The problem can be formulated as: given the traffic profile \( \lambda \) and the renewable energy profile \( P_H \), adjust the resource allocation and the grid energy input \( (n, P_G) \) to minimize the average grid power consumption while satisfying the weighted outage probability
\[
\min_{(n, P_G)} \frac{\sum_{t=1}^T L_t P_{G,t}}{\sum_{t=1}^T L_t} \tag{13}\]
\[
\text{s.t. } \sum_{t=1}^T \omega_t p_{\text{out},t} \leq p_{\text{target}}, \tag{14}\]
where the outage probability \( p_{\text{out},t} \) is expressed as (7), and the weighting factor \( \omega_t \), which satisfies \( \sum_{t=1}^T \omega_t = 1 \), reflects the system sensitivity to the outage probability in each slot. The weighting factor allows for the case that users may require higher QoS at some particular times of the day.

B. Optimal DP Solution

The optimal solution for problem (13) with constraint (14) can be found by exhaustive search through all possible policies, which however, is not practical due to its high complexity. The DP approach [5], which divides the whole problem into simple per-stage sub-problems, is a candidate approach to find the optimal policy. We consider the following unconstrained optimization problem with a weighted combination of the power consumption and the blocking probability
\[
\min_{(n, P_G)} \frac{\sum_{t=1}^T L_t P_{G,t}}{\sum_{t=1}^T L_t} + \beta \sum_{t=1}^T \omega_t p_{\text{out},t}, \tag{15}\]
where the factor \( \beta > 0 \) plays the role of the Lagrangian multiplier and indicates the relative importance of the outage probability over the average grid power consumption.

The DP algorithm contains three key components: state, action and cost function. In the problem (15), the state is the remaining energy \( E_{B,t} \) in the battery. The action is the number of active resource blocks \( n_t \) and the grid power \( P_{G,t} \).

The per-stage cost is the weighted combination of the average grid power and the outage probability, denoted as a function of the current action and state
\[
e_t(n_t, P_{G,t}, E_{B,t}) = \frac{L_t P_{G,t}}{\sum_{t=1}^T L_t} + \beta \omega_t p_{\text{out},t}. \tag{16}\]

The DP algorithm breaks the original problem down into sub-problems with respect to the stage, and solves them recursively. The cost-to-go function is defined recursively as
\[
J_T(n_T, P_{G,T}, E_{B,T}) = \min_{n_T, P_{G,T}} \left\{ c_T(n_T, P_{G,T}, E_{B,T}), \right. \tag{17}\]
\[
J_t(n_t, P_{G,t}) = \min_{n_t, P_{G,t}} \left\{ c_t(n_t, P_{G,t}, E_{B,t}) + J_{t+1}(E_{B,t+1}) \right\}. \tag{18}\]
where \( t < T \), which denotes the minimum cost of the sub-problem with slot \( t \) as its initial stage. Performing a backward induction of the cost-to-go functions (17) and (18) from time slot \( T \) to slot 1, we can obtain the minimum cost equal to \( J_1(0) \). Correspondingly, the average grid power and the weighted outage probability are denoted as \( P_{G,0}^* \) and \( p_{\text{out},0}^* \), respectively, which means \( J_1(0) = P_{G,0}^* + \beta p_{\text{out},0}^* \). Due to the optimality of \( J_1(0) \), \( P_{G,0}^* \) must be the minimum average grid power to guarantee that the outage probability is no more than \( p_{\text{out},0}^* \). We denote \( P_{G,0}^* \) as the minimum average grid power such that the outage probability does not exceed \( p_{\text{out}} \). Hence, we have \( P_{G,0}^* = P_{G,0}^* \). By adjusting the value of \( \beta \) and solving the corresponding problem (15), we can find a set of points for the function \( P_{G,0}^*(p_{\text{out}}) \). Joining these points forms a curve indicating the lower bound of grid power consumption for the target outage. Any achievable pair of grid power consumption and outage probability must be above the curve. Notice that we may not find \( P_{G,0}^*(p_{\text{out}}) \) for all \( p_{\text{out}} \) as there is resource block on-off state alternating. Hence, if a point for a given \( p_{\text{out}} \) can be found by setting appropriate value of \( \beta \), the optimal solution for the original problem (13) is found. Otherwise, we can just get a suboptimal result by adopting the policy corresponding to the point with the largest outage probability less than \( p_{\text{out}} \).
C. Heuristic Algorithms

Besides the optimal DP algorithm, the following heuristic policies are proposed for comparison:

- **Maximum resource block utilization.** In this policy, all the blocks are activated for transmission, i.e., \( n_t = N \) for all \( t \). It can be considered as a baseline.

- **Traffic-aware resource block utilization.** Based on the intuition that higher traffic intensity requires more wireless resources, we propose the policy that the number of activated resource blocks is set proportional to the traffic intensity, i.e.,

\[
    n_t = \min\{N, \lceil \eta_1 \lambda_t N \rceil \}, \quad \eta_1 > 0 \tag{19}
\]

where \( \lceil x \rceil \) is the minimal integer no smaller than \( x \).

- **Joint traffic-energy-aware resource block utilization.** As the outages are caused not only by lack of wireless resources, but also by the lack of power, the power budget should be taken into consideration. In this case, the number of active resource blocks is also proportional to the available power besides traffic intensity:

\[
    n_t = \min\{N, \lceil \eta_2 \lambda_t \frac{E_{B,t} + E_{G,t} + L_t P_{H,t}}{\sum_{k=t}^{T} L_k (P_0 + \Delta P_{T})} N \rceil \}, \tag{20}
\]

where \( \eta_2 > 0 \), and the grid energy \( E_{G,t} \) is evolved as \( E_{G,t+1} = \max\{0, E_{G,t} - L_t P_{G,t} \} \). Note that \( P_0 + \Delta P_{T} \) in the denominator is for normalization.

The grid power \( P_{G,t} \) in all these policies is obtained in the same way as follows. Given the average grid power \( P_{Gave} \), the grid energy budget is initialized as \( E_{G,1} = \sum_{t=1}^{T} L_t P_{Gave} \). We get \( P_{G,t} = \min\{ \frac{E_{G,t}}{L_t}, \max\{0, P_0 + \frac{\Delta P_{T}}{L_t} - P_{H,t} \} \} \), i.e., the grid power is used to satisfy the power requirement as long as it is available.

V. Numerical Results

We examine the performance of the proposed algorithms by numerical simulations. We adopt the energy consumption model of the macro BS from the EARTH project [13], and the radio frequency model from 3GPP LTE [14]. In the macro-cell scenario, we have \( P_0 = 712.2 \text{W}, \Delta P = 15.96, P_T = P_{\max} = 40 \text{W}, \text{R} = 1000 \text{m} \). The sleep mode power is \( P_S = 0.5 \text{W} \) corresponding to a deep sleep. We consider the \( W_0 = 10 \text{MHz} \) bandwidth with the number of sub-carriers \( N = 600 \). The path-loss is \( PL_{\text{dB}} = 34.5 + 35 \log_{10}(l) \), and the noise power density is \(-174 \text{dBm/Hz}\).

We first study the relationship between outage probability and active resource blocks under different parameter settings, as shown in Fig. 1. The elements of the parameter vector are traffic intensity \( \lambda \), user’s rate requirement \( r_0 \) (in Mbps) and average input power \( P_{\text{in}} \) (in Watt), respectively. It can be seen that when the energy supply is sufficient for the BS operation, \( Pr(P_1 = P_T) = 1 \), outages only happen when the required rate is not achievable. However, when the energy is insufficient, outages due to energy unavailability will gradually become the main factor. As a result, the outage probability decreases as the number of active resource blocks increases, and then goes up again once the energy input is not enough to support the required number of active resource blocks. Hence, there is a minimum outage probability working point as shown by the star on each curve.

Then the performance of the grid power minimization algorithm is evaluated with a given traffic profile and energy arrival statistics in a day. The rate requirement is set to \( r_0 = 0.5 \text{Mbps} \), and the traffic profile and renewable energy harvesting profile are taken from [13] and [15], respectively, as shown in Fig. 2. We set \( T = 24 \), and the length of each slot is \( L_t = L = 1 \text{h} \). The algorithms run from 9:00 to the same time of the next day. The traffic profile is represented by the ratio \( \phi_t \) of the maximum traffic intensity \( \lambda_{\max} = 10 \), i.e., \( \lambda_t = \phi_t \lambda_{\max} \).

The tradeoff between average outage probability \( \omega_t = 1/T \) and grid energy consumption for different policies is depicted in Fig. 3. It can be seen that the proposed DP based algorithm is the optimal solution. In comparison, the maximum resource utilization algorithm performs the worst over a wide range of grid power inputs (less than 700Watt). In this algorithm, a high level of outages occur as the algorithm requires the base station always to operate at maximum power. The traffic-aware algorithm greatly improves the performance compared with maximum resource block utilization. By adjust-
choosing proper better than the traffic-aware policy in almost all conditions by
In addition, the joint traffic-energy-aware policy performs
(\eta_1 = 0.12) is near optimal for the high grid energy input regime.
In this simulation, we set \( \omega \) for the low grid power input regime, and that with large
obtain different outage profile. Specifically, the algorithm tends
to reduce the outage probability in a high traffic load regime
if the corresponding weighting factor is large (\( \eta_1 = 0.12 \)).
It can be seen that by adjusting the weighting factor, we can
reduce the outage probability in a high traffic load regime, the grid energy consumption can be
reduced by designing proper weighting factors.

VI. CONCLUSION

We have analyzed the relation between grid energy input and outage probability in single-cell energy harvesting wireless systems. The proposed DP algorithm finds the optimal tradeoff between the average grid power and the outage probability. For a 2% average outage probability, the DP reduces by 50% the grid energy consumption for the maximum resource utilization policy and by 30% for the traffic-aware policy with \( \eta_1 = 0.12 \).
Among the heuristic algorithms, the joint traffic-energy-aware resource allocation policy performs closest to the optimal one. In addition, proper choices of the outage probability weights can further reduce the energy consumption. Future work will extend the results to the multi-cell scenario.

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