On the Online Minimization of Completion Time in an Energy Harvesting System

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Abstract—This paper considers a single-transmitter energy harvesting system and looks into the problem of completion time minimization from the worst-case point of view. In offline study, an optimal algorithm [1] is given to yield the minimum completion time of transmission. However, for online algorithms, the randomness of future energy arrivals adds to the difficulty of scheduling, thus the offline minimum completion time cannot always be reached. This leads to the question "What is the deterministic performance bound of online algorithms compared to the offline optimum". By a game-theoretic method, this paper shows that with an infinite-sized battery, there exist several algorithms that guarantee a completion time no more than the twice of its offline counterpart for all possible energy arrivals, and more importantly that the ratio of two cannot be further reduced. This property is of great significance especially when reliability is valued in the system.

I. Introduction

In this paper, we consider a communication system where the transmitter is powered by an energy harvesting module with a sufficiently large battery. According to the current size of buffered data and the battery state, the transmitter determines the transmit power at each time slot in order to minimize the transmission completion time, defined as the number of time slots between the arrival of data request at the transmitter and the completion of transmission. Note that the queueing delay in the buffer is not considered. The channel is assumed to be time-invariant, so that the rate-power function of the channel does not change with time.

It is nontrivial and challenging to investigate the power allocation policy of an energy harvesting communication system, due to the randomness of energy arrivals [2]. Specially, energy-delay tradeoff in conventional wireless networks has been well studied. But the completion time, which directly adds to the delay in energy harvesting systems, requires more attention.

For the offline case, where all the information of future energy arrivals is known in advance, the optimal power allocation schemes for different scenarios have been investigated. Optimal algorithms for transmission completion time minimization with an infinite-sized battery and a finite battery are given in [1] and [3], respectively. A directional water-filling algorithm [4] and a similar staircase water-filling algorithm [5] are proposed for throughput maximization with fading channels. The impact of circuit power consumption is considered in [6] and [7].

As for the online settings, where only the historical information is available to the transmitter, the design of optimal policy by dynamic programming, which requires a known probability distribution on the energy arrival [8], is discussed [4]. Several sub-optimal algorithms for throughput maximization are also proposed [4], but none of them offers performance guarantee. Moreover, online throughput maximization in finite-horizon is discussed [9]. For static channel with Bernoulli energy arrivals, a power allocation algorithm for long-term average throughput maximization is proposed, leading to a mean throughput within a constant gap from the optimum [10]. Outage minimization with energy harvesting transmitters and receivers is studied and several online algorithms are established [11] [12].

The most related work to ours is [13], in which the worst case ratio between the completion time under online algorithms and the offline optimum is analyzed. This paper shows that there is a lower bound of 1.38 for the competitive ratio of online algorithms in a single-user additive white Gaussian noise channel, and 1.356 for a two-user multiple access channel. Additionally, a 2-competitive lazy algorithm is proposed. However, the tight bound of competitive ratio remains unknown.

In contrast to average-case analysis, which is widely applied in communication systems, a deterministic evaluation provides important information about the overall performance of an algorithm. Especially in communication systems where transmissions are vital to the receiver and are triggered infrequently, a worst-case performance bound is of interest in order to insure a reliable transmission and a satisfying quality of service, additional to mean-value evaluation. Such applications include wireless sensor networks for malfunction detection and alarming, automatic braking systems, and other safety-sensitive scenarios. Besides, while average-case performance depends on the modeling of random events and the extraction of system parameters, worst-case analysis offers a global view of the system and is robust to the uncertainty of an stochastic process. It is also worth mentioning that since simulations cannot efficiently trace all the possible energy arrivals for an energy harvesting system, a theoretical analysis is crucial for obtaining the performance bound in this problem.

To the best of our knowledge, there has not been a known performance bound for online completion time minimization with energy harvesting transmitters, especially for the case of general energy arrivals and general rate-power function. In this paper, the basic problem with time-invariant channels and infinite-sized battery is characterized, and the problem of com-

pletion time minimization is investigated from a competitive-analytic perspective. This problem is thereafter formulated as a two-person zero-sum game, which has a value of 2, to obtain the tight lower bound of competitive ratio of online algorithms. The main contributions of this paper are summarized as follows:

1) An online case with general rate-power functions is studied. The results in this paper do not rely on an explicit rate-power mapping, instead of which only a few basic properties about the function are assumed. The most distinctive assumption about the function is the concavity, which is commonly encountered.
2) The infimum of competitive ratio of online algorithms is proved to be 2. In other words, for online algorithms, the largest ratio between the actual completion time and the offline optimum is no less than 2. This indicates that both the two algorithms proposed in this work are optimal from the worst-case point of view.
3) Two intuitive algorithms are proposed. It is proved that an algorithm named Linger-On-then-Keep-Invariable (`LOKI`) achieves the minimum competitive ratio. An improved version of `LOKI` is shown. Comparison between the two algorithms and the generalized lazy online (GLO) algorithms proposed in [13] is given.

The outline of the rest of the paper is as follows. The next section explicitly describes the system model, presents basic assumptions and offers some preliminary concepts of competitive analysis. The main results is first presented in Section III. Section IV introduces the online algorithm `LOKI`. Several properties of energy arrival process and online policies, with which the action sets of the nature and the designer are reduced, are presented in Section V. Then, the two-person zero-sum game formulation of this problem is established in Section VI, based on which the infimum of competitive ratio is obtained. Properties of three optimal algorithms are discussed in Section VII. The last section concludes this paper.

II. SYSTEM MODEL

Consider a time-slotted energy harvesting system as shown in Fig. 1. At the first time slot $t = 0$, the transmitter is requested to deliver a packet of D bits to the receiver. The only available power source of the transmitter is an energy harvesting module that harvests energy from renewable sources. At each time slot $t \in \mathbb{N}$, the harvested energy, denoted by $e(t)$, is stored in an infinite-sized battery. With complete unawareness of all the information about the energy harvesting process $E = \{e(t), t \in \mathbb{N}\}$, including the statistical distribution or the average rate of the harvesting dynamics, the transmitter looks for an online algorithm π to manage currently available energy in the battery, determines the transmit power $p(t)$ at each slot $t \in \mathbb{N}_{>0}$, and delivers the packet to the receiver as quickly as possible. Here, we assume that the energy consumption for auxiliary actions at the transmitter side, such as calculation and decision-making, is negligible compared with transmission over the air, so that only the transmit power is considered in the sequel.
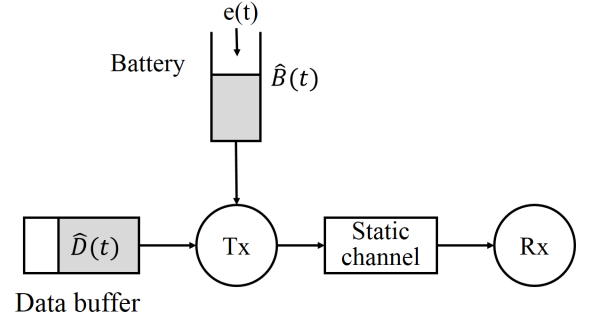

Fig. 1. Energy harvesting system with completely random energy arrivals and an infinite-sized battery

Due to causality requirements on energy, the allocated transmit power $p(t)$ can not exceed currently available energy $\hat{B}(t)$ in the battery, i.e.,

$$p(t) \leq \hat{B}(t), \tag{1}$$

where

$$\hat{B}(t) = \sum_{\tau=0}^{t-1} e(\tau) - \sum_{\tau=1}^{t-1} p(\tau). \tag{2}$$

After the power allocation, the transmitter sends out a signal going through a time-invariant channel with a rate-power mapping $g: \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ from transmit power to channel rate, or the number of bits transmitted in a slot. The rate function $g(\cdot)$ is assumed to have the following properties:

1) $g(\cdot)$ is monotonically increasing;
2) $g(\cdot)$ is strictly concave;
3) $g(0) = 0$, and $g(\cdot)$ is unbounded;
4) There exists a constant α, such that

$$\alpha = \lim_{x \to 0} g'(x).$$

Some remarks are as follows. In the first two assumptions, monotonicity and concavity of the rate function together indicate the law of diminishing marginal utility, which refers to the decrease in marginal increment of channel rate as transmit power rises. This property suggests a uniform allocation of energy throughout different slots in order to maximize short-term throughputs. The third assumption admits the possibility that an extremely high transmit power produces desirable channel rate, while the fourth assumption forbids the exchange of high latency for excessively low energy consumption by the finiteness of α, which is a scale coefficient between the channel rate and the transmit power when transmitting at an extremely low power level.

Here are some examples supporting the rationality of the above assumptions about channel rate. For static additive white Gaussian noise (AWGN) channels with transmit power P and noise variance N, the classic Shannon's function $C = \frac{1}{2}\log(1 + P/N)$ satisfies all of the four conditions. Further, note that the single receiver in Fig. 1 does not indicate a single-input-single-output (SISO) channel, where the transmitter is

dedicated to a single receiver. In fact, the analysis in this paper applies to all the channel types with a qualified sum-rate function, such as the one of two-user multiple-access channel with uncoordinated coding described in [13], which multiplies the channel rate and the transmit power in Shannon's function by a scale factor and does not affect the concavity. Similar extensions can be made to multiple-receiver scenarios.

For brevity, we also denote the size of unfinished task at the beginning of slot t by $\hat{D}(t)$, such that

$$\hat{D}(t) = D - \sum_{\tau=1}^{t-1} g(p(\tau)), \forall t \in \mathbb{N}_{>0}. \tag{3}$$

A. Offline Optimum

Given energy arrival profile E and data size D, the offline minimum completion time is the solution to the following program:

$$\text{minimize} \quad T \tag{4}$$

$$\text{subject to} \quad \sum_{\tau=1}^{t} p(\tau) \leq \sum_{\tau=0}^{t-1} e(\tau), \forall t \in \{1, 2 \cdots, T\}, \tag{5}$$

$$\sum_{\tau=1}^{T} g(p(\tau)) \geq D. \tag{6}$$

Constraint (5), which is equivalent to (1), stands for the causality constraint in energy before the transmission is completed. Upon the termination of transmission, at least D bits are delivered to the receiver, which explains (6).

An offline completion time minimization problem is similar to finite-horizon throughput maximization, which can be written as

$$\text{maximize} \quad \sum_{\tau=1}^{T} g(p(t)) \tag{7}$$

$$\text{subject to} \quad \sum_{\tau=1}^{t} p(\tau) \leq \sum_{\tau=0}^{t-1} e(\tau), \forall t \in \{1, 2 \cdots, T\}.$$

The optimal result of completion time minimization is identical with the smallest time length T in which the optimal throughput of (7) is greater than or equal to requested data size D. Besides, Theorem 2 in [3] summarizes some insights into the relationship between completion time minimization and throughput maximization. It states that the power allocation schemes of the two problems are identical for correspondent data size (throughput) D and time length T. This property suggests that a throughput maximization algorithm can be easily transformed into a completion time minimization algorithm, and an optimal offline algorithm is given in [3].

B. Online Settings

Consider the online case where only historical energy arrivals are known to the transmitter. In every time slot t, the transmitter observes the size $\hat{D}(t)$ of the remaining data in the buffer, previous energy arrivals E_{t-1}, and previous allocations p_{t-1}, where

$$E_t = (e(0), e(1), \cdots, e(t))$$

and

$$p_t = (p(1), p(2), \cdots, p(t)),$$

and selects a certain power level for transmission. Our goal is to minimize the completion time T of the transmission of data with size D, i.e., to find an online algorithm π, which depends only on p_{t-1}, E_{t-1}, and D, such that algorithm π leads to a relatively small completion time $T(\pi, E)$ satisfying $\hat{D}(T(\pi, E) + 1) = 0$ and (1)–(3).

The deficiency of future information brings a dilemma for the transmitter. On one hand, the optimal offline algorithm suggests a relatively conservative power allocation scheme, i.e., spending less energy at the very beginning of transmission and increasing the transmit power at certain energy arrivals points, which might leads to a long transmission time. On the other hand, an aggressive algorithm could result in the shortage of available energy for subsequent transmissions, even possibly in failing to deliver the required D bits of data.

C. Competitive Analysis

Since neither prior probability distributions nor statistical characteristics of energy arrival process E is assumed, a distributional or average-case analysis is impractical. Here, we adopt competitive analysis and study completion time minimization from a worst-case viewpoint to evaluate the performance of an online algorithm. Relevant definitions in [14] and some notations are as follows:

Denoting the offline result of (4) under a certain energy arrival process $E = \{e(t), t \in \mathbb{N}\}$ as $T_{off}(E)$, an online algorithm π is said to be strictly c-competitive if for all the energy arrivals E that the offline completion time $T_{off}(E)$ is finite, the completion time $T(\pi, E)$ is upper bounded as

$$T(\pi, E) \leq c \cdot T_{off}(E).$$

The *competitive ratio* of π is defined as the infimum of all the c that π is called c-competitive. Equivalently, the competitive ratio of π is denoted as $\lambda(\pi)$ and the following equation holds:

$$\lambda(\pi) = \sup_E \frac{T(\pi, E)}{T_{off}(E)}. \tag{8}$$

Further, an online algorithm π is called *competitive* if it attains a finite competitive ratio, i.e., $\lambda(\pi) < +\infty$.

Since online results are always no better than its offline optimum counterparts, we have

$$\lambda(\pi) \geq 1, \forall \pi,$$

which means that the competitive ratio of online algorithms are lower bounded. Our goal is to find the tight bound of competitive ratio of online algorithms for completion time minimization with a general rate-power function $g(\cdot)$:

$$\inf_\pi \lambda(\pi) = \inf_\pi \sup_E \frac{T(\pi, E)}{T_{off}(E)}.$$

Further, an online algorithm π^* is called *optimal* if it attains the minimum competitive ratio $\inf_\pi \lambda(\pi)$.

III. MAIN RESULT

Here we summarize the main conclusion of this paper in the following theorem.

Theorem 1: With an infinite-sized battery and a time-invariant rate-power function satisfying the four assumptions in Section II, the infimum of competitive ratio of online algorithms for completion time minimization is 2:

$$\inf_{\pi} \sup_{E} \frac{T(\pi, E)}{T_{off}(E)} = 2. \quad (9)$$

Other than seeking an optimal algorithm after obtaining the lower bound, we first propose a simple 2-competitive algorithm in the next section, then prove the main theorem via this algorithm in Section VI.

IV. 2-COMPETITIVE ONLINE ALGORITHM

Here we propose a Linger-On-then-Keep-Invariable(LOKI) algorithm, which is simpler but also 2-competitive.

Algorithm 1 LOKI for completion time minimization

Input: D;
1: $t \leftarrow 0$;
2: $B \leftarrow 0$;
3: **repeat**
4:     Harvest an energy packet of size $e(t)$;
5:     $B \leftarrow B + e(t)$;
6:     $t \leftarrow t + 1$;
7: **until** $t \cdot g(\frac{B}{t}) \geq D$;
8: $P_0 \leftarrow \frac{B}{t}$;
9: **repeat**
10:     Transmit with power level P_0 at slot t;
11:     $t \leftarrow t + 1$
12: **until** transmission completed.

Algorithm LOKI is rather intuitive. It postpones the transmission until adequate energy and time, with which the data can be delivered, is gained. With energy arrival process E, the allocated transmit power of each time slot can be given as

$$p(t) = \begin{cases} 0, & t = 1, 2, \cdots, T_1 - 1; \\ P_0, & t = T_1, T_1 + 1, \cdots, T_{\texttt{LOKI}}(E), \end{cases}$$

where T_1 is the time slot in which the transmission begins, i.e., the smallest integer that satisfies

$$T_1 \cdot g\left(\frac{\sum_{\tau=0}^{T_1-1} e(\tau)}{T_1}\right) \geq D,$$

and

$$P_0 = \frac{\sum_{\tau=0}^{T_1-1} e(\tau)}{T_1}.$$

It is easy to show that LOKI does not violate the causality constraint in (1). Since the transmission can be completed with a power level of P_0 in T_1 slots, the second repeat-until iteration terminates in T_1 slots, therefore the energy accumulated before $t = T_1$ is enough for the transmission.

Although LOKI might seem slightly pessimistic and conservative, it actually attains a simple form as well as a good performance. Additionally, it will show great importance in subsequent analysis on the infimum of competitive ratio in Section VI. In this section, we first look into the characteristics of LOKI by introducing the following theorem.

Lemma 1: LOKI attains a competitive ratio of 2.

Proof: First, we prove that the offline minimum completion time T_{off} is no less than T_1 by *reductio ad absurdum*.

According to the algorithm, T_1 is the result of the following program:

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & t \cdot g\left(\frac{\sum_{\tau=0}^{t-1} e(\tau)}{t}\right) \geq D, \\ & t \in \mathbb{N}. \end{aligned}$$

Thus, we get

$$t \cdot g\left(\frac{\sum_{\tau=0}^{t-1} e(\tau)}{t}\right) < D, \ \forall t = 1, 2, \cdots, T_1 - 1. \quad (10)$$

Assuming that the offline optimal solution is $\{p^*(t)\}_{t>0}$ and it leads to an offline completion time of T_{off}, we have

$$\sum_{\tau=1}^{T_{off}} g(p^*(\tau)) \geq D \quad (11)$$

with one of the causal constraints

$$\sum_{\tau=1}^{T_{off}} p^*(\tau) \leq \sum_{\tau=0}^{T_{off}-1} e(\tau).$$

Due to the concavity and monotonicity of rate-power function $g(\cdot)$, we have

$$\sum_{\tau=1}^{T_{off}} g(p^*(\tau)) \leq T_{off} g\left(\frac{\sum_{\tau=1}^{T_{off}} p^*(\tau)}{T_{off}}\right) \leq T_{off} g\left(\frac{\sum_{\tau=0}^{T_{off}-1} e(\tau)}{T_{off}}\right).$$

According to (10), if $T_{off} < T_1$, then

$$\sum_{\tau=1}^{T_{off}} g(p^*(\tau)) \leq T_{off} g\left(\frac{\sum_{\tau=0}^{T_{off}-1} e(\tau)}{T_{off}}\right) < D,$$

which contradicts (11). Therefore, the following inequality holds:

$$T_{off} \geq T_1. \quad (12)$$

Second, we show that the competitive ratio of algorithm LOKI is no larger than 2. Since

$$T_1 \cdot g(P_0) = T_1 \cdot g\left(\frac{\sum_{\tau=0}^{T_1-1} e(\tau)}{T_1}\right) \geq D,$$

the transmitter stays silent for the first $T_1 - 1$ slots and needs at most T_1 time slots after slot T_1 to fulfill the transmission, so we have

$$T_{\texttt{LOKI}}(E) \leq 2T_1 - 1 \leq 2T_{off} - 1.$$

Thus, the competitive ratio of the LOKI algorithm satisfies

$$\lambda_{\texttt{LOKI}} = \sup_{E} \frac{T_{\texttt{LOKI}}(E)}{T_{off}(E)} \leq 2.$$

Third, we show that the competitive ratio is no less than 2. Consider an energy arrival process $E = \{e(t), t \in \mathbb{N}\}$ as follows:

$$e(t) = \begin{cases} g^{-1}\left(\dfrac{D}{T_0}\right), & t = 0, 1, \cdots, T_0 - 1; \\ 0, & \text{otherwise.} \end{cases}$$

The optimal offline strategy is to transmit with a constant power level $g^{-1}\left(\frac{D}{T_0}\right)$ during the first T_0 slots, and

$$T_{off}(E) = T_0, \tag{13}$$

while $\texttt{LOKI}$ suggests to transmit with the same power level from slot T_0, and

$$T_{\texttt{LOKI}}(E) = 2T_0 - 1. \tag{14}$$

Dividing both sides of (14) by the corresponding side of (13), and take supremum over $T_0 \in \mathbb{N}$, we have

$$\lambda_{\texttt{LOKI}} \geq \sup_{T_0} \frac{T_{\texttt{LOKI}}(E)}{T_{off}(E)} = 2.$$

Therefore, the competitive ratio of algorithm $\texttt{LOKI}$ is 2. ∎

V. Reducing the action set

Up to now, constraints on energy arrival processes are hardly made, and online algorithms are required to follow only the causality constraint of energy. Before introducing the game-theoretic formulation of completion time minimization in Section VI, we first look into the possible energy arrival processes and power allocation algorithms to reduce potential candidates of both sides, such that the problem can be simplified. Hereafter, we refer to the set of all the potential energy arrival processes as the action set of nature, and the set of all candidate power allocation algorithms as the action set of the transmitter. The reduced action sets of both nature and the transmitter are presented in the following two subsections.

A. The Action Set of Nature

Due to the fourth assumption about the rate function $g(\cdot)$, a disproportionate amount of energy might result in the failure of transmission, even in an offline setting. In order to make this problem reasonable, the total energy harvested must be adequately large. To obtain the action set $\mathcal{I}$ of nature, first we introduce the following lemma.

Lemma 2: For an energy arrival process $E = \{e(t), t \in \mathbb{N}\}$, the transmission of data with size D can be completed in a finite length of time if, and only if, $D < \alpha \sum_{\tau=0}^{+\infty} e(\tau)$.

Proof: *Sufficiency*: For any power allocation scheme $\{p(t)\}_{t>0}$, the size $D(t)$ of data transmitted in the first t slots satisfies

$$\begin{aligned} D(t) &= \sum_{\tau=1}^{t} g(p(\tau)) \\ &< t \cdot g\left(\frac{\sum_{\tau=1}^{t} p(\tau)}{t}\right) \\ &\leq t \cdot g\left(\frac{\sum_{\tau=0}^{+\infty} e(\tau)}{t}\right). \end{aligned}$$

The first inequality holds because of the concavity of $g(\cdot)$, and the second is due to the causality constraint of energy. Taking the limit of the last expression as t approaches infinity, we obtain

$$\begin{aligned} D(t) &\leq \lim_{t \to +\infty} t \cdot g\left(\frac{\sum_{\tau=0}^{+\infty} e(\tau)}{t}\right) \\ &= \lim_{x \to 0} g'(x) \sum_{\tau=0}^{+\infty} e(\tau) \\ &= \alpha \sum_{\tau=0}^{+\infty} e(\tau). \end{aligned}$$

Thus, any transmission to be fulfilled in a finite number of slots must have a data size strictly less than $\alpha \sum_{\tau=0}^{+\infty} e(\tau)$.

Necessity: Let $\epsilon = \frac{-D + \alpha \sum_{\tau=0}^{+\infty} e(\tau)}{2} > 0$. Since $D + 2\epsilon = \alpha \sum_{\tau=0}^{+\infty} e(\tau)$, there exists a positive integer T_2, such that

$$\alpha \sum_{\tau=0}^{T_2-1} e(\tau) > D + \epsilon. \tag{15}$$

Meanwhile, because

$$\lim_{t \to +\infty} t \cdot g\left(\frac{\sum_{\tau=0}^{T_2-1} e(\tau)}{t}\right) = \alpha \sum_{\tau=0}^{T_2-1} e(\tau),$$

there exists another positive integer T_3, such that

$$T_3 \cdot g\left(\frac{\sum_{\tau=0}^{T_2-1} e(\tau)}{T_3}\right) > \alpha \sum_{\tau=0}^{T_2-1} e(\tau) - \epsilon. \tag{16}$$

Allocating the energy as

$$\begin{cases} p(t) = 0, & t = 1, \cdots, T_2 - 1; \\ p(t) = \dfrac{\sum_{\tau=0}^{T_2-1} e(\tau)}{T_3}, & t = T_2, \cdots, T_2 + T_3 - 1, \end{cases}$$

and substituting (15) into (16), the total size of data that can be transmitted in $T_2 + T_3 - 1$ slots is

$$T_3 \cdot g\left(\frac{\sum_{\tau=0}^{T_2-1} e(\tau)}{T_3}\right) > D.$$

Thus, the transmission can be fulfilled in a finite time interval. ∎

Here, we define the action set $\mathcal{I}$ of the nature to be the set of all energy arrival processes under which it is possible to fulfill a transmission in an finite time length. Therefore, the action set can be written as

$$\mathcal{I} = \left\{ \{e(t), t \in \mathbb{N}\} \mid \sum_{\tau=0}^{+\infty} e(\tau) > \frac{D}{\alpha} \right\}.$$

B. The Action Set of the Transmitter

Combined with the causality constraint, the action set Π of the minimizer is defined to be the set of all competitive and causal strategies, i.e.,

$$\Pi = \left\{ \pi \mid \lambda(\pi) < +\infty, p(t) \leq \hat{B}(t), \forall t \right\}. \tag{17}$$

Denoting Algorithm 1 as π_{LOKI}, since the competitive ratio $\lambda(\pi_{\text{LOKI}}) = 2$, π_{LOKI} is both causal and competitive, thus $\pi_{\text{LOKI}} \in \Pi$. Here we give an evident lemma.

Lemma 3: (Reliability guarantee) For any strategy $\pi \in \Pi$, the transmit power $p(t)$ at each time slot t satisfies

$$\alpha p(t) - g(p(t)) < \max(\alpha \hat{B}(t) - \hat{D}(t), 0). \quad (18)$$

Proof: In order to guarantee the completion of transmission in a finite time interval, according to lemma 2, the transmit power must follow the constraint

$$\hat{D}(t) - g(p(t)) < \alpha \left[\hat{B}(t) - p(t) \right], \forall t \in \mathbb{N}_{>0}, \quad (19)$$

as long as there is enough energy, i.e.,

$$\hat{D}(t) < \alpha \hat{B}(t).$$

The left-hand side of (19) equals the residual data at the beginning of slot $t + 1$, and the right-hand side the upper bound of the size of data that can be transmitted, assuming no further energy arrivals after t. Adding $\alpha p - \hat{D}(t)$ to both sides of (19), (18) holds when $\hat{D}(t) < \alpha \hat{B}(t)$.

If $\hat{D}(t) \geq \alpha \hat{B}(t)$, any power allocation $p(t) > 0$ might lead to the failure in completion, thus bring a infinite competitive ratio. Therefore, we have $p(t) = 0$, and (18) is also satisfied. ∎

From lemma 3, the following corollary can be easily obtained, and we will omit the proof.

Corollary 1: If $\alpha \sum_{\tau=0}^{t_0-1} e(\tau) < D$, then $\forall \pi \in \Pi, \forall t \leq t_0, p(t) = 0$.

Corollary 1 gives a hint about the structure of Π. It indicates that for any competitive and causal strategy, the transmitter would remain silent before the total amount of harvested energy reaches a level associated with the data size D.

VI. A TWO-PERSON ZERO-SUM GAME

From a game-theoretic view, the original problem, which focuses on finding an online algorithm with a minimum competitive ratio, forms a two-person zero-sum game in pure strategy. The kernel function of the game is given by

$$J(\pi, E) = \frac{T(\pi, E)}{T_{off}(E)} \quad (20)$$

and we have

$$\lambda(\pi) = \sup_E J(\pi, E). \quad (21)$$

In this game, the transmitter, or the strategy designer, decides on the online algorithm and acts as the minimizer, while nature seeks an energy arrival process E to maximize the kernel function.

In accordance to [15], the upper value of this two-person zero-sum infinite game is defined by

$$\overline{V} = \inf_\pi \sup_E J(\pi, E), \quad (22)$$

while the lower value is

$$\underline{V} = \sup_E \inf_\pi J(\pi, E). \quad (23)$$

If the upper value equals the lower value, i.e., $V = \overline{V} = \underline{V}$, then the game is said to have a value of V.

To illustrate the relation between the two-person zero-sum game and the competitive analysis of completion time minimisation, we substitute (21) into (22), and get

$$\overline{V} = \inf_\pi \lambda(\pi),$$

which suggests that the upper value of the game equals exactly the infimum of the competitive ratio of online algorithms. Therefore, if the game has a value, it can be simpler to obtain the tight lower bound of competitive ratio.

However, it is evident that the original game does not have a value, since $\underline{V} = 1$ and $\overline{V} > 1$. But with the reduced action set $\Pi \times \mathcal{I}$, the game has a value of 2, i.e.,

$$\sup_{E \in \mathcal{I}} \inf_{\pi \in \Pi} J(\pi, E) = \inf_{\pi \in \Pi} \sup_{E \in \mathcal{I}} J(\pi, E) = 2. \quad (24)$$

The proof is given in the rest of the section.

Lemma 4: For any positive value ϵ, this game has an ϵ saddle point. More explicitly, $\forall \epsilon > 0, \exists (\pi^*, E^*) \in \Pi \times \mathcal{I}$, s.t. $\forall (\pi, E) \in \Pi \times \mathcal{I}$,

$$J(\pi^*, E) - \epsilon \leq J(\pi^*, E^*) \leq J(\pi, E^*) + \epsilon \quad (25)$$

Proof: Construct an energy arrival process $E^*_{T_0} = \{e^*_{T_0}(t), t \in \mathbb{N}\}$ as

$$e^*_{T_0}(t) = \begin{cases} g^{-1}\left(\frac{D}{T_0}\right), & t = 0, 1, \cdots, T_0 - 1; \\ 0, & \text{otherwise,} \end{cases}$$

and let $l(T_0)$ represents the integer satisfying

$$\frac{D}{\alpha g^{-1}\left(\frac{D}{T_0}\right)} - 1 \leq l(T_0) < \frac{D}{\alpha g^{-1}\left(\frac{D}{T_0}\right)}. \quad (26)$$

Dividing (26) by T_0, the inequality becomes

$$\frac{\frac{D}{T_0}}{\alpha g^{-1}\left(\frac{D}{T_0}\right)} - \frac{1}{T_0} \leq \frac{l(T_0)}{T_0} < \frac{\frac{D}{T_0}}{\alpha g^{-1}\left(\frac{D}{T_0}\right)}. \quad (27)$$

Taking limit as T_0 approaches infinity at both sides of (27), we have

$$\lim_{T_0 \to +\infty} \frac{\frac{D}{T_0}}{\alpha g^{-1}\left(\frac{D}{T_0}\right)} = \lim_{T_0 \to +\infty} \frac{l(T_0)}{T_0} = 1.$$

Therefore,

$$\lim_{T_0 \to +\infty} \frac{l(T_0)}{T_0} = 1. \quad (28)$$

It is obvious that the optimal offline strategy under $E^*_{T_0}$ is to transmit with a constant power level $g^{-1}\left(\frac{D}{T_0}\right)$ at the first T_0 slots, and

$$T_{off}(E^*_{T_0}) = T_0.$$

For online strategies, according to Corollary 1, any strategy $\pi \in \Pi$ will not choose to transmit before $l(T_0) + 1$, since

$$\sum_{\tau=0}^{l(T_0)-1} e^*_{T_0}(\tau) = l(T_0) g^{-1}\left(\frac{D}{T_0}\right) < \frac{D}{\alpha}.$$

Also notice that all algorithms need at least T_0 slots for transmission, thus

$$T(\pi, E_{T_0}^*) \geq l(T_0) + T_0, \forall \pi \in \Pi. \tag{29}$$

Dividing both sides of (29) by offline optimum T_0, we get

$$J(\pi, E_{T_0}^*) \geq 1 + \frac{l(T_0)}{T_0}.$$

Since in (28) the limit of $\frac{l(T_0)}{T_0}$ is 1 as T_0 goes to infinity, we have that $\forall \epsilon > 0, \exists N_0(\epsilon) \in \mathbb{N}^+$, s.t. $\forall T_0 \geq N_0(\epsilon)$,

$$\frac{l(T_0)}{T_0} \geq 1 - \epsilon,$$

therefore

$$J(\pi, E_{T_0}^*) \geq 2 - \epsilon.$$

Recall algorithm LOKI proposed in section IV. We have

$$J(\pi_{\texttt{LOKI}}, E) \leq \lambda(\pi_{\texttt{LOKI}}) = 2, \ \forall E \in \mathcal{I}.$$

For any $\epsilon > 0$, let $T_0 = \max\left(N_0(\epsilon), \lceil\frac{1}{\epsilon}\rceil\right)$, and $\pi^* = \pi_{\texttt{LOKI}}$, then $\forall \pi \in \Pi$

$$\begin{aligned} J(\pi, E_{T_0}^*) + \epsilon &\geq& 2 \\ &\geq& J(\pi^*, E_{T_0}^*). \end{aligned}$$

And $\forall E \in \mathcal{I}$,

$$\begin{aligned} J(\pi^*, E) - \epsilon &\leq& 2 - \epsilon \\ &\leq& 2 - \frac{1}{T_0} \\ &\leq& J(\pi^*, E_{T_0}^*). \end{aligned}$$

Thus, inequality (25) holds. ∎

According to Theorem 4.1 in [15], a two-person zero-sum game has an ϵ saddle point for every positive value ϵ if, and only if, this game has a finite value, and the value equals the limit of the Cauchy sequence $\{J_{\epsilon_k}(\pi^*, E^*)\}_k$ with $\epsilon_{k+1} < \epsilon_k$ and $\lim_{k \to +\infty} \epsilon_k = 0$, where $J_{\epsilon_k}(\pi^*, E^*)$ represents the middle item of (25) with $\epsilon = \epsilon_k$. Therefore, we have the main result in Section III, and Theorem 1 is proved below.

Proof: First, we prove that the two-person zero-sum game has a value of 2 in pure strategy. Applying the ϵ saddle points found in the proof of Lemma 4, the limit of the Cauchy sequence is

$$\begin{aligned} \lim_{k \to +\infty} J_{\epsilon_k}(\pi^*, E^*) &=& \lim_{\epsilon \to 0} J(\pi^*, E_{T_0(\epsilon)}^*) \\ &=& \lim_{T_0 \to +\infty} \frac{2T_0 - 1}{T_0} \\ &=& 2. \end{aligned}$$

Therefore, we have

$$\sup_{E \in \mathcal{I}} \inf_{\pi \in \Pi} J(\pi, E) = \inf_{\pi \in \Pi} \sup_{E \in \mathcal{I}} J(\pi, E) = 2$$

holds.

Substituting the kernel function (20) into (24), the equation is given by $\inf_{\pi \in \Pi} \sup_{E \in \mathcal{I}} \frac{T(\pi, E)}{T_{off}(E)} = 2$, which is quite similar to the left-hand side of (9). The only difference is
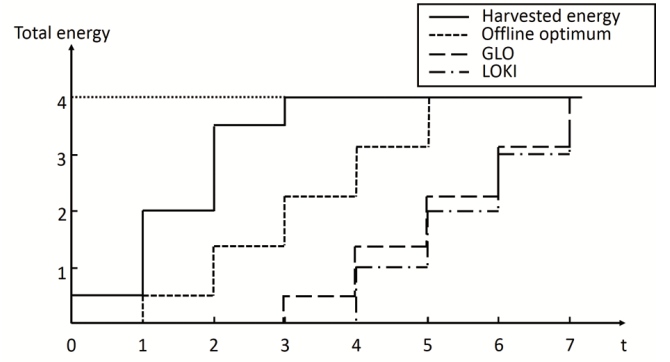

Fig. 2. The energy allocation policies of offline optimum and LOKI under a certain energy arrival process

that the action sets in the zero-sum game are restricted to all competitive and causal algorithms and all reasonable energy arrival processes. Thus, any causal algorithms that are not in Π must have a competitive ratio of infinity, so the above equation can be extended as

$$\inf_{\pi} \sup_{E \in \mathcal{I}} \frac{T(\pi, E)}{T_{off}(E)} = 2.$$

With all energy arrival processes that might lead to a finite completion time included in $\mathcal{I}$, it is sufficient to conclude that the infimum of competitive ratio of online algorithms for completion time minimization is 2. ∎

VII. DISCUSSION

An energy arrival instance and its allocation schemes under the offline optimum, LOKI, and GLO in [13] are illustrated in Fig. 2. The solid line represents the total amount of harvested energy up to the corresponding time slot, while the short-dashed line stands for the strategy that leads to the shortest completion time, the dot-dashed line for the policy suggested by LOKI and the long-dashed line for the policy by GLO. In the instance, we assume that the rate-power function is $g(p) = \log(1 + p)$ and the data size $D = 4$, thus the transmission can be completed with four units of energy and four time slots, i.e., $4 \times g(1) = D$. Since the available energy at the first slot is less than one unit, the transmission can not be completed in four slots, and the optimal strategy is to use up all the initial energy at the first slot, then set the transmit power to an constant level during the following four slots, which gives a total completion time of five slots. With algorithm LOKI, the transmitter waits for three slots, and then allocates the harvested energy equally to the following four slots. Under GLO algorithm, the transmission does not start until the third slot as well because of energy shortage. It first transmits with 0.5 unit of energy at the third slot and increases the power to 0.875 at the fourth slot. For both LOKI and GLO algorithm, the completion time is seven slots, which is less than the twice of the offline minimum.

Theorem 1 gives the tight lower bound of the competitive ratio. It is proved that both the GLO algorithm in [13] and

`LOKI` reach the minimum competitive ratio of 2, which suggests that they are optimal from the worst-case viewpoint. With GLO implemented, the controller needs to set the transmit power to a reasonably higher level at each energy arrival, which might require excessive computational steps, thus bring unacceptable delay as well as energy consumption. Instead, `LOKI` shows better potential for facilitating computation by distributing these overhead expenses to the slots before the actual transmission begins. Besides, for the case where energy is harvested frequently and the harvested energy in each time slot is relatively low, `LOKI` greatly reduces the calculation cost.

Further, several other optimal algorithms can be established through similar method. For example, a simple modification can be made to `LOKI` by replacing the constant transmit power with time-adaptive ones, as described in Algorithm 2. This algorithm increases the transmit power at each energy arrival after the transmission begins, while `LOKI` only utilizes the former accumulated energy once the transmitter starts to deliver the data. It is obvious that this algorithm is always 'ahead' of `LOKI`, thus optimal. However, more computation steps are required at the transmitter's side. Although computational cost is not considered in this work, the tradeoff between completion time and actual calculation costs exists among the optimal algorithms in real systems.

Algorithm 2 Another 2-competitive online algorithm for completion time minimization

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Input: D;
  1: $t \leftarrow 0$;
  2: $B \leftarrow 0$;
  3: **repeat**
  4:     Harvest an energy packet of size $e(t)$;
  5:     $B \leftarrow B + e(t)$;
  6:     $t \leftarrow t + 1$;
  7: **until** $t \cdot g(\frac{B}{t}) \geq D$;
  8: **repeat**
  9:     **if** $e(t)$ amount of energy is harvested **then**
 10:        $B \leftarrow B + e(t)$;
 11:        P_0 equals the largest real number p satisfying $\lfloor \frac{B}{p} \rfloor \cdot g(p) \geq D$;
 12:     **end if**
 13:     Transmit with power level P_0 at slot t;
 14:     $D \leftarrow D - g(P_0)$;
 15:     $B \leftarrow B - P_0$;
 16:     $t \leftarrow t + 1$;
 17: **until** transmission completed.

VIII. CONCLUSIONS

In this work, we study the delivery of information from an energy harvesting transmitter with an infinite-sized battery to other nodes through a time-invariant channel with a concave rate-power function. The problem of completion time minimization is investigated through competitive analysis. By a game-theoretic method, the infimum of competitive ratio of online algorithms for this problem is proved to be 2. In other words, for any online algorithm, the supremum of the ratio between its completion time and the one of the offline optimum over all possible energy arrivals is no less than 2. Two new algorithms for this problem, under which the transmitter stays silent until enough energy and time is accumulated, is proposed. The comparison of optimal algorithms is given, and the characteristics of these algorithms is discussed. It is shown that the lower bound of 2 can be achieved with relatively small computational overhead.

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