A Practical Channel Allocation Scheme Based on the Weighted Conflict Graph in Heterogeneous Networks

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***Abstract*—Heterogeneous Networks (HetNets), where Low Power Nodes (LPN) are deployed under the coverage of Macro Base Stations (MBS), are promising to boost the spectrum efficiency per unit area. However, this hierarchical architecture also brings new problems, like severe inter-cell interference. The randomly deployed LPNs, and the co-existence of the cross-tier and intra-tier interference make it challenging to design the effective frequency reuse schemes, which can have significant influences on the user experience and the network capacity. In this paper, we explore the frequency reuse problem in HetNets for interference mitigation. Firstly, Partial Spectrum Reuse (PSR) scheme is adopted to mitigate the cross-tier interference, where MBSs can use all available spectrums while LPNs can only use part of the channels based on their traffic load. Then, the channel allocation scheme is further optimized to mitigate the intra-tier interference. As the channel allocation problem is NP-hard, we propose a greedy channel allocation scheme based on a weighted conflict graph, where cross-tier and intra-tier interference are both considered. In addition, our method is practical for channel allocation in real systems. Simulation results show the SINR of cell edge users and the ergodic SINR can be both improved by 3dB as much with our greedy method compared with the random channel allocation scheme.**

I. Introduction

Heterogeneous Networks (HetNet), which is a multi-tier cellular network with different types of Base Stations (BS) co-deployed, is expected to be the dominant scenario in 4G and 5G eras [1]. With more cells sharing the same bandwidth, HetNets is promising to boost the spectrum efficiency. However, as the frequency reuse factor increases, so does the interference. In addition, with macro base stations (MBS) cells overlapped with Low Power Nodes (LPN) (like micro BSs, pico BSs and femto BSs) cells, more users may find themselves at cell edges suffering from severe interference. Therefore, interference has become an important factor that limits the system capacity and user experience.

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In fact, the interference mitigation problem in homogeneous networks has been intensively studied. The main idea is to further divide the cells into geographical sectors and allocate orthogonal channels to neighboring sectors [2]. However, the interference problem in HetNets is much more challenging compared with the homogeneous networks. Firstly, besides the intra-tier interference, there also exists cross-tier interference. Secondly, even the intra-tier interference (interference caused by the BSs of the same tier/type) of the LPNs is still complicated to handle for the random deployment. The interference management method for homogeneous networks (such as [2]) is not suitable for the LPNs.

To solve the interference problem in HetNets, many studies have been done. By applying the idea of cognitive radio, the QoS of the macro users were guaranteed by whilst the LPNs used channels as secondary users [4] - [6]. Frequency reuse schemes based on geographic position were proposed in [7] - [10] to mitigate intra-tier interference. In addition, cross-tier and intra-tier interference were both considered in [11] - [13], where LPNs chose sub-channels based on the detected channel information. However, the decision is conducted only with the local information, which may degrade the global performance.

As LPNs usually have much fewer traffic load compared with the MBS, it is reasonable to allocated only part of the channels to each LPNs while the MBSs can use all the spectrum (named as Partial Spectrum Reuse (PSR) [3]). PSR scheme can mitigate the inter-tier interference and hence improve network performance. In addition, the analytical result of the optimal proportion of channels allocated to the LPNs, which minimized the network power consumption, was obtained in a two-tier HetNet [3]. However, the LPNs were assumed to choose their channels randomly in [3]. Therefore, the detailed channel allocation scheme between LPNs has yet to be designed, which is our focus in this paper. Firstly, we formulate the channel allocation problem as an optimization problem which turns out to be NP-hard. To solve this problem, we propose a greedy method based

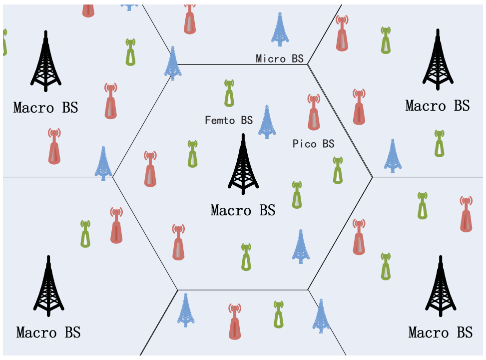


Fig. 1: General scenario of HetNets

on the weighted conflict graph, where intra-tier and cross-tier interference are both reflected. Simulations are conducted to evaluate the proposed method, and the results show that our greedy method is very close to the exhaustive search. In addition, the ergodic SINR and the SINR of cell edge users can be both improved by 3dB compared with the random channel allocation scheme. Furthermore, our proposed method has only linear complexity to the number of LPNs and available channels, which can be easily applied to real systems.

The rest of the paper is organized as follows. In Section II, we review the related works. Then, the channel allocation problem is described in Section III. In Section IV, the weighted conflict graph is introduced, based on which a greedy algorithm is proposed to solve the NP-hard channel allocation problem. In Section V, simulations are conducted to evaluate the performance of the proposed method. Finally, some conclusions are drawn in Section VI.

II. Related Work

In this section, we introduce the existing works about interference mitigation in HetNets and the application of conflict graph on interference avoidance.

Location-based method Location-based channel allocation for LPNs was studied in [7] - [10]. In [7], a two-tier network was considered. The macro cells were further divided into multiple regions, and orthogonal channels were allocated to neighboring regions for intra-tier interference mitigation between the MBSs. Meanwhile, the LPN users and macro users in the same region were allocated with orthogonal channels to avoid the cross-tier interference. In [8], the bandwidth used by the LPNs was adjusted according to their distance to the MBSs. However, the intra-tier interference of the LPNs was ignored in these studies. [9] and [10] considered to reserve some particular bandwidth for the *cell edge* users, but this may decrease the spectrum efficiency.

Sensing-based method In [11] - [13], LPNs chose sub-bands in a decentralized way. [11] only considered the intra-tier interference between the LPNs while [12] [13] also considered the cross-tier interference. In [11] [12], the LPNs chose channels based on the measured channel condition information. Channel allocation was conducted in a random way in [13]. Another idea is based on the idea of cognitive radio. The authors in [4] - [6] treated the LPNs as the secondary users, and the QoS of the macro users was guaranteed in priority. But the QoS of the LPN users can not be satisfied for the unreliable opportunistic transmission.

Conflict graph method Interference mitigation on conflict graph has also been extensively explored in existing studies. In [16] [17], the multi-channel multi-radio case in wireless networks was studied, and multiple dimension graphs were used to solve the problem by linear programming method. Based on conflict graph, channel-assignment methods were proposed in [14] [15]. A homogeneous multi-cell OFDMA system was considered in [14], where the cells were further divided into geographical clusters and the interference relationship was modeled by a conflict graph. In addition, the indoor case of femto BSs was analyzed in [15]. However, these studies only considered the intra-tier interference among the LPNs. Besides, protocol interference model was adopted, which means the interference between two BSs was modeled to as 0-1 integer.

In this paper, we design a channel allocation scheme based on a weighted conflicted graph, where physical interference model is adopted. Besides cross-tier and intra-tier interference are both considered. To the authors' knowledge, there is no such work in the existing studies.

III. System Model and Problem Formulation

A. System Model

We consider the general case of HetNets, where MBSs and different types of LPNs co-exist (shown in Fig.1). The MBSs are assumed to be regularly deployed in hexagonal cells. In addition, the LPNs are assumed to be randomly deployed and we do not limit the distribution of the LPNs.

Let $\mathcal{C} = \{1, ..., C\}$ denote the set of available channels, $\mathcal{M} = \{1, ...M\}$ denote the set of MBSs, and $\mathcal{L} = \{1, ...L\}$ denote the set of LPNs. In fact, the traffic load of a typical LPN is generally much lighter than that of a MBS. Therefore, we apply PSR scheme in [3] to mitigate the cross-tier interference, where the MBSs can use all channels available, but LPN-l can only use B_l ($B_l \leq C$) channels based on their traffic load. B_l is treated as known parameters, whose design is out of the scope of this paper.

With PSR to mitigate the cross-tier interference, our problem is to optimize the channel allocation of the LPNs. Denote $\mathcal{I} = [I_{lc}]_{L\times C}$ channel allocation schemes:

$$I_{lc} = \begin{cases} 1, & \text{if channel-}c\text{ is allocated to LPN-}l \\ 0, & \text{otherwise} \end{cases}, \quad (1)$$

where $l \in \mathcal{L}$ and $c \in \mathcal{C}$.

As for the user association policy, we assume that users will choose the MBS or LPN which offers the maximal received signal to interference ratio (SINR) for best service experience. The received SINR of user-u, who is assumed to be associated with i (which can be a MBS or a LPN) and use channel-c, is given by

$$\gamma_{uic} = \begin{cases} \frac{P_i G_{iu}}{\sum\limits_{j \in \mathcal{M}, j \neq i} P_j G_{ju} I_{jc} + \sum\limits_{k \in \mathcal{L}} P_k G_{ku} I_{kc} + \sigma^2}, & i \in \mathcal{M} \\ \frac{P_i G_{iu}}{\sum\limits_{j \in \mathcal{M}} P_j G_{ju} I_{jc} + \sum\limits_{k \in \mathcal{L}, j \neq i} P_k G_{ku} I_{kc} + \sigma^2}, & i \in \mathcal{L} \end{cases}, \tag{2}$$

where G_{iu} is the channel gain between user-u and server i, P_i is the transmit power of i, $\mathcal{M}$ and $\mathcal{L}$ denote the sets of MBSs and LPNs respectively, and σ^2 is the noise power. When adaptive modulation and coding are used, the spectrum efficiency of received SINR γ_{uic} can be approximated as [18]:

$$C(\gamma_{uic}) = \log_2(1 + \beta\gamma_{uic}), \tag{3}$$

where $\beta = -1.5\lg(5\varepsilon)$ is a constant related to the bit error rate requirement ε. Without losing generality, assume each user is allocated with one channel. Then the maximal data rate of user-u is given by

$$R_{uic} = W_{\text{ch}} C(\gamma_{uic}), \tag{4}$$

where W_{ch} denotes the bandwidth of one channel. As users are assumed to be uniformly distributed, then the average data rate for users associated with server i is given by:

$$\mathbb{E}_u\{R_{uic}\} = \int\limits_{u \in \mathcal{A}_i} W_{\text{ch}} C(\gamma_{uic}) f_u(u) \mathrm{d}u, \tag{5}$$

where $\mathcal{A}_i$ denotes the coverage area of i, $f_u(u)$ denotes the probability distribution function of the location of user-u.

B. Problem Formulation

For fairness reason, the channel allocation problem is formulated as

$$\begin{aligned} \max_{\mathcal{I}} \quad & \min_{c \in \mathcal{C}, i \in (\mathcal{M} \bigcup \mathcal{L})} \mathbb{E}_u\{R_{uic}\} \\ s.t. \quad & \sum_{c \in \mathcal{C}} I_{lc} = B_l, l \in \mathcal{L} \end{aligned} \tag{6}$$

where B_l is the number of channels required by LPN-l. Specially, the physical meaning of the target function is to maximize the minimal average data rate per user of all servers (including MBSs and LPNs) among all channels. Thus, the optimization of channel allocation has been modeled as a nonlinear mixed integer programming problem. Unfortunately, this kind of problems is NP-hard. In addition, as the small cells are deployed randomly, even the explicit expression of the target function in (6) can not be obtained, which makes the problem more complicated.

IV. Greedy Channel Allocation Based on the Weighted Conflict Graph

In this section, we simplify (6) to interference mitigation problem, as the data rate of the users depends on the interference they received. In addition, we propose a greedy method to solve the problem based on the weighted conflict graph.

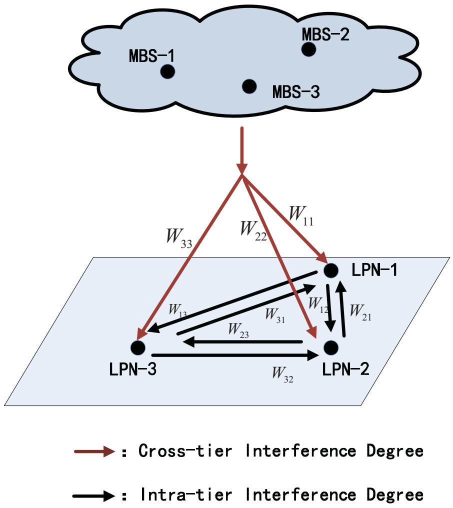


Fig. 2: An example of conflict graph

In fact, different channel allocation schemes with the same B_l do not influence the average data rate of the MBSs, as MBSs occupy all the available channels. Besides, the transmit power of the LPNs is much lower than the MBSs. Therefore, the users of the LPNs generally have worse service experience compared with the MBS users. Therefore, we should focus on the interference suffered by LPNs.

A. Weighted Conflict Graph

We construct a weighted conflict graph $G = (V, E, W)$ to represent the physical interference suffered by the LPNs, which includes both intra-tier interference between LPNs and the cross-interference caused by MBSs. Each node $v_i \in V$ denotes one LPN, and edge $e_{ij}(i \neq j)$ denotes that LPN-i causes interference to LPN-j. Note that G is a directed complete graph according to the physical model, since there always exists interference between any two LPNs. Besides, the weight of each edge $W_{ij}(i \neq j)$ is used to reflect the physical interference caused by LPN-i to LPN-j. For simplicity, W_{ij} can be defined as the *interference degree*, which is given by:

$$W_{ij} = \frac{P_i}{P_j}\left(\frac{d_{ij}}{D}\right)^{-\alpha}, i \neq j, \tag{7}$$

where α is the path loss factor, P_i and P_j is the transmit power of server i and server j respectively, and d_{ij} is the corresponding distance. Here, D is called *reference distance*, which is an empirical value. Generally, two LPNs are considered to have severe interference if their distance is smaller than D.

Specifically, we define W_{ii} as the cross-tier interference suffered by LPN-i, which is given by

$$W_{ii} = \sum_{m \in \mathcal{M}} \frac{P_m}{P_i}\left(\frac{d_{mi}}{D}\right)^{-\alpha}, i \in \mathcal{L}. \tag{8}$$

Therefore, the interference degree matrix $W = [W_{ij}]_{L \times L}$ can reflects both intra-tier and cross-tier interference suffered by each LPN. Thus, the channel allocation can be designed simply based on the conflict graph $G = (V, E, W)$. An example of our weighted conflict graph is given in Fig.2, where three LPNs and three MBSs are considered. Our innovation is that the cross-tier interference is added to graph G by redefining W_{ii}, which has no physical meanings in the traditional conflict graph.

B. Problem Simplification

Based on the weighted conflict graph constructed, problem (6) can be simplified as follows:

$$\begin{aligned} \min_{\mathcal{I}} \quad & \left\{ \max_{c\in\mathcal{C}, i\in\mathcal{L}} \left\{ \sum_{j\in(\mathcal{M}\bigcup\mathcal{L})} W_{ji} I_{jc} I_{ic} \right\}\right\} \\ s.t. \quad & \sum_{c\in\mathcal{C}} I_{lc} = B_l, l \in \mathcal{L} \end{aligned} \tag{9}$$

where $W_{ji}(j \neq i)$ is the interference degree between LPN-j and LPN-i, and W_{ii} is the cross-tier interference for LPN-i, which can be obtained by the conflict graph. The physical meaning of the objective function is to minimize the maximal interference suffered by all the LPNs among all channels. Actually, the interference suffered by LPN-i indicates the channel condition of users in LPN-i on average. Therefore, the average data rate of the users can be reflected by the in the objective function if no intra-cell power control or dynamic bandwidth allocation is considered. Therefore, (9) and (6) have analogous function. Furthermore, the validation of this simplification will be given in Section V.

C. Greedy Channel Allocation

According to Eq.9, we are trying to minimize the maximal interference suffered by the LPNs. To achieve this goal, we design our channel allocation algorithm based on two principles:

1) the LPN with the highest interference degree chooses its channels first;
2) When a LPN chooses its channels, the channels with lower interference are chosen, such that $\max_c \hat{W}_c$ is minimized.

Then, we propose our greedy channel allocation algorithm, whose flow chart has been shown in Fig.3, where the node set $\bar{V}$ of conflict graph $\bar{G}$ denotes the LPNs which have been allocated with channels. As $\sum_{i\in V} W_{in}$ denotes the total interference received by LPN-n from all the LPNs in conflict graph G, LPN-n^* is the LPN suffered from the most severe interference among all the LPNs who have not been allocated with channels. Then, we allocate its required number of channels (B_{n^*}). During this process, we first calculate the maximal interference suffered by all the LPNs under channel-c if LPN-n^* also joins in, which is denoted as $\mathbf{W}(c)$ in the flow chart. Then, B_{n^*} channels with smaller value of $\mathbf{W}(c)$ are allocated to LPN-n^*. With this method, the increase of the maximal interference will be minimized after the channels are allocated with LPN-n^*. For the first

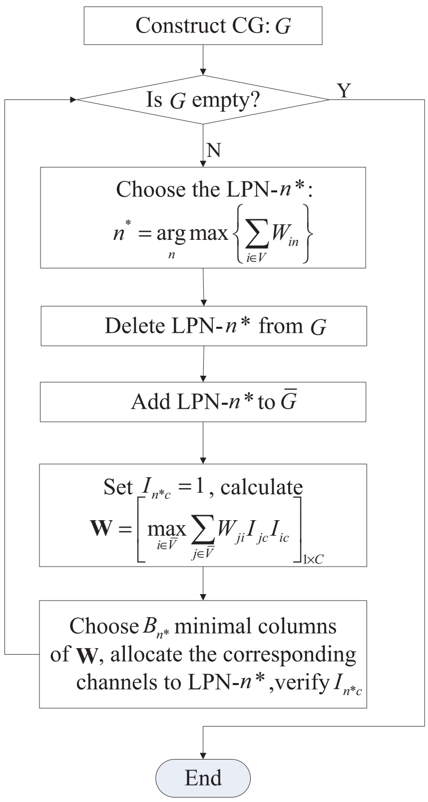


Fig. 3: Greedy algorithm based on conflict graph

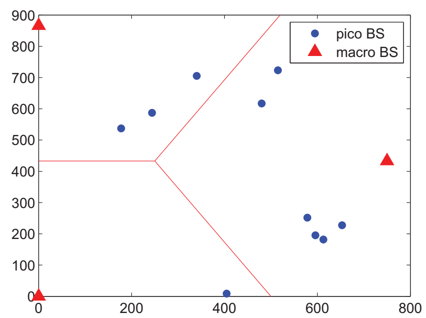


Fig. 4: Simulation scenario

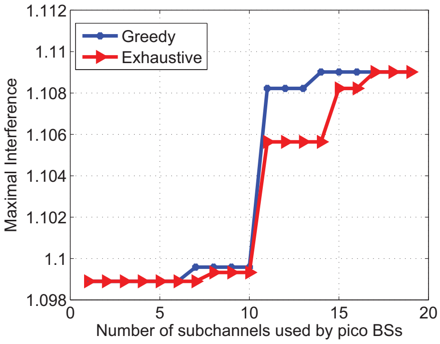
LPN, it will choose channels in a random way as all the channels have the same interference.

The complexity of this greedy algorithm is $O(LC)$, where L is the number of LPNs to be arranged and C is the number of available channels.

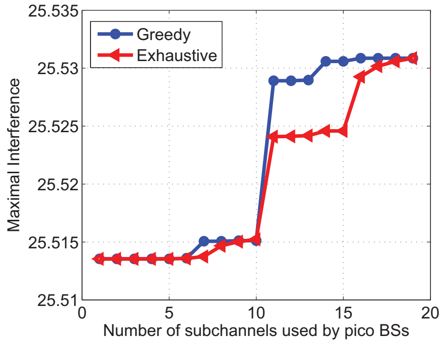
V. Performance Evaluation

In this section, we conduct simulation to evaluate our scheme from two perspectives:

1) The performance loss of our greedy algorithm compared with the optimal solution for problem (9);
2) The validation of the problem simplification from problem (6) to problem (9).

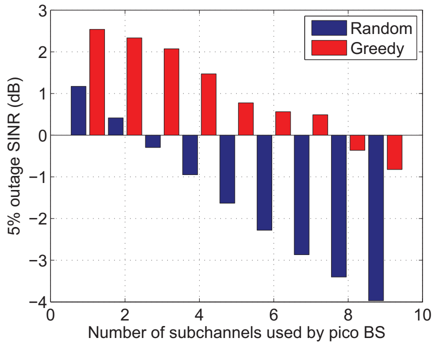


(a) 3 pico BSs

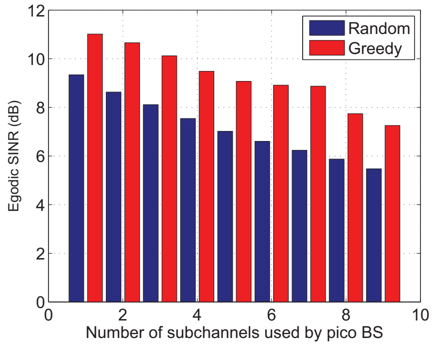


(b) 4 pico BSs

Fig. 5: Maximal interference of the networks on average



(a) SINR for cell edge users



(b) Ergodic SINR

Fig. 6: Statistic SINR of users

A. Optimality Evaluation

To evaluate our greedy algorithm, we compare it with the optimal solution of (9) obtained by exhaustive search. A two-tier HetNet consists of 3 MBS and several pico BSs (PBS) shown as Fig.4 is considered. In the simulation, the coverage radius of the MBSs is set to 500m, the path loss factor is $\alpha = 4$, the number of available channels is $C = 10$, reference distance $D = 50$m, and the transmit power of the MBSs and PBSs is $P_{\mathrm{M}} = 20$W and $P_{\mathrm{p}} = 1$W, respectively. The locations of the MBSs are fixed at the center of the hexagonal cells. Considering that the PBSs are usually deployed at the cell edge of the MBSs to enhance the coverage and capacity in real systems, we set some *forbidden regions* where no PBSs are deployed. More specifically, each forbidden region is a circle of radius 200m and centered at one MBS. Then, the distance between the PBSs and the MBSs are no smaller than 200m.

We set the number of PBSs in the HetNets as 3 or 4 to conduct exhaustive search. PBSs are considered to be uniformly distributed in the area out of the forbidden regions. Furthermore, all PBSs require the same number of channels. Fig.5 shows the numerical results of problem (9) obtained by averaging 1000 random network topology samples. The X-axis is the number of channels required by each PBS, and the Y-axis is the value of the target function of problem (9), i.e., the maximal interference of the network. As shown in the figure, the results of our greedy algorithm are very close to the exhaustive search method. Specially, our algorithm will only cause 3% extra loss in both cases, which indicates the effectiveness of the greedy method.

The curves in Fig.5 are not smooth. Firstly, the channel allocation scheme is modeled as integer programming. More importantly, the maximal interference of the network does not always increase with B_i (may remains the same), which depends on the detailed network topology and the relationship between B_i and C. To make it clear, we show two examples.

(1) $1 \leq B_i \leq C/N$. Intuitively, the optimal channel allocation method is to allocated orthogonal channels to all LPNs, which is feasible as the total required number of channels by all the LPNs is no larger than the number of available channels ($\sum_{i \in \mathcal{L}} B_i \leq C$). Under this condition, there is no intra-tier interference. Therefore, the maximal interference is independent of B_i when $1 \leq B_i \leq C/N$. Therefore, the interference of the network does not increase with B_i.

(2) $B_i = \lfloor \frac{1}{2} C \rfloor + 1$. For most cases, the network interference increase as B_i approaches to $\lfloor \frac{1}{2}C \rfloor + 1$. Therefore, there exists an obvious increase of the network interference when each PBS requires 11 channels shown in Fig.5. The reason is that any two PBSs have to share common channels when the number of required channel B_i satisfies $B_i > \frac{1}{2}C$. Assume PBS-1 and PBS-2 have maximal inter-cell interference without lose of generality. If $B_i \leq \frac{1}{2}C$, there always exists a feasible channel allocation scheme where PBS-1 and PBS-2 use orthogonal channels for interference avoidance. But PBS-1 and PBS-2 have to share at least one channel when $B_i > \frac{1}{2}C$. In addition, the situation will be even worse when BS-1 and BS-2 are close to each other. Therefore, the maximal interference in the network will increase as B_i approaches to $\frac{1}{2}C + 1$ for most case.

The above examples can explain why the interference curves do not increase with B_i smoothly. In addition, this also shows insights on the design of B_i. For example, $B_i = 13$ is a better choice than $B_i = 11$, since $B_i = 13$ can increase the spatial spectrum reuse without increase the network interference, shown in Fig.5.

B. System Performance

To validate (9), we simulate the SINR of users, compared with the results of random channel allocation [3]. The simulation scenario is shown as Fig.4, a HetNets consists of three MBSs and 10 PBSs. Users are assumed to be uniformly distributed in the network and choose the BS which offers the maximal received SINR. Fig.6 gives both the 5% outage SINR and the ergodic SINR of 100000 users samples with random locations.

The 5% outage SINR reflects the service quality for the cell edge users who are facing severe inter-cell interference. With our proposed greedy channel allocation algorithm, the 5% outage SINR has been improved by about 3dB as much (Fig.6a), which means the greedy method can effectively mitigate the inter-cell interference. The ergodic SINR reflects the service performance of most users. In Fig.6b, the ergodic SINR has also been improved by 3dB as much, therefore, the network capacity can also be improved by our method.

VI. Conclusion

In this paper, we have proposed a practical channel allocation scheme based on the weighted conflict graph in HetNets. Specially, our weighted conflict graph adopted the physical interference model, which reflects both intra-tier and cross-tier interference. Simulation results show the effectiveness of our algorithm. Both the ergodic SINR and the SINR of the cell edge users can be improved by about 3dB, comparing with the random channel allocation. One big advantage of our scheme is the linear complexity. Future work should consider the design of spectrum reuse factor when users are non-uniformly distributed.

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