

# Minimum Power Consumption of a Base Station with Large-scale Antenna Array

Zhiyuan Jiang, Sheng Zhou, and Zhisheng Niu

Tsinghua National Laboratory for Information Science and Technology (TNList)

Dept. of Electronic Engineering, Tsinghua University, Beijing, China

E-mail: jiang-zy10@mails.tsinghua.edu.cn, {sheng.zhou, niuzhs}@tsinghua.edu.cn

**Abstract**—In this paper we consider the minimum base station (BS) power consumption given the sum rate requirement in large-scale multiple-input-multiple-output (MIMO) systems. A single cell with an  $M_{\text{tot}}$ -antenna BS and  $N$  single-antenna users is considered. The BS power consumption consists of two parts: The part accounting for the total transmit power and the part proportional to the number of active antennas. Specifically, closed-form approximations (CFAs) of the optimal transmit power and optimal number of active antennas are derived when the sum rate requirement is high. A CFA of the ergodic sum capacity upper bound for the downlink broadcast channel is also given.

## I. INTRODUCTION

Recent years have witnessed the rapid growth of the information and communication technology (ICT) industry, which is becoming a significant part of the world energy consumption. Particularly cellular networks are one of the main energy consumers in the ICT industry. It is reported that the base stations (BSs) are responsible for 80% of the total energy consumption of cellular networks. Therefore, reducing the energy consumption of BSs is the key to support future development of cellular networks.

The concept of large-scale multiple-input-multiple-output (MIMO) system is drawing much academic attention and becoming an emerging technology, which scales up the number of BS antennas by an order of magnitude compared to current state-of-the-art. It not only brings capacity gain but also reduces the power consumption dramatically [1]. The downlink channel of large-scale MIMO systems is in nature the MIMO broadcast channel. The capacity of BC has been studied intensively in the literature. In [2], the capacity region of BC is shown to coincide with the dirty-paper coding achievable rate region. The sum capacity is expressed by a max-min problem in [3] and an uplink-downlink duality is also found. Nonetheless, little is known for the ergodic capacity of BC, especially when the channel matrix is non-identically distributed (NID). On the other hand, the optimal number of active radio-frequency (RF) chains is studied in [4], however only simulation results are given. And the authors in [5] considers the optimal active RF chains in a point-to-point MIMO system.

This paper analyzes the minimum power consumption of one BS with a large-scale antenna array given the user sum

rate requirement. Specifically, we consider the communication between a BS with  $M_{\text{tot}}$  available antennas and  $N$  single-antenna users. However only  $M$  out of  $M_{\text{tot}}$  antennas are active to support the rate requirement in the cell. In this paper, the BS power consumption model is obtained from the EARTH project [6]

$$P_{\text{BS}} = P_0 + MP_s + \Delta_P P_t, \quad (1)$$

where  $P_0$  is the static BS power consumption,  $P_s$  is the BS power consumption of each antenna and the corresponding RF chain,  $\Delta_P$  is the transmit power coefficient related to power amplifier (PA) efficiency, and  $P_t$  is the total downlink transmit power of all the active antennas. Among these coefficients,  $P_s$  and  $\Delta_P$  can be obtained from [6].  $P_0$  is regarded as a constant and does not affect the optimization, therefore it is omitted in the analysis hereinafter. The typical values of these parameters are shown in Table I. Based on this model, the rate-dependent parts of the BS power consumption not only involve  $P_t$ , which is directly related to the sum capacity, but also include the number of active antennas  $M$ , which also has a great impact on the sum capacity. The problem considered in this paper is: Given the sum rate requirement and  $N$  users' locations in the cell, what is the minimum base station (BS) power consumption, i.e.

$$\begin{aligned} &\text{Minimize} && P_{\text{BS}}, \\ &s.t. && R(P_{\text{BS}}) \geq R_{\text{req}}, \\ &&& MP_s + \Delta_P P_t \leq P_{\text{BS}} \end{aligned} \quad (2)$$

where  $M$  and  $P_t$  are jointly optimized. We assume the channel state information (CSI) is perfectly known to the BS and users prior to data transmission<sup>1</sup>.

The main contribution of this work is deriving the closed-form approximations (CFAs) of the optimal number of active antennas  $M^*$  and optimal transmit power  $P_t^*$ , along with the minimum BS power consumption in large-scale MIMO systems when the system sum rate is high. A CFA of the downlink broadcast channel (BC) ergodic sum capacity upper bound is derived to facilitate the analysis. All CFAs are verified numerically.

This work is sponsored in part by the National Basic Research Program of China (2012CB316001), the Nature Science Foundation of China (61201191, 60925002, 61021001), and Hitachi Ltd.

<sup>1</sup>In practice, this can be done using training sequences and channel estimation methods [7].

$$C^*(P_t, M, \tilde{V}) \approx \begin{cases} M \log\left(\frac{M}{M-N}\right) + N \log\left(\frac{M-N}{N} \frac{P_t \tilde{V}}{\sigma_n^2}\right) - N \log(e) & \text{if } M > N, \\ N \log\left(\frac{P_t \tilde{V}}{\sigma_n^2}\right) - N \log(e) & \text{if } M = N, \end{cases} \quad (3)$$

## II. CHANNEL MODEL

The channel is characterized as

$$\mathbf{y} = \sqrt{\frac{P_t}{M}} \mathbf{G} \mathbf{Q} \mathbf{x} + \mathbf{z}, \quad (4)$$

where  $\mathbf{x} \in \mathcal{C}^M$  denotes the transmit signal of  $M$  active antennas at the BS with unit variance entries.  $\mathbf{y} \in \mathcal{C}^N$  denotes the received signal of  $N$  single-antenna users.  $\mathbf{z} \sim \mathcal{C}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$  is the additive Gaussian noise.  $\mathbf{Q} \in \mathcal{C}^{N \times M}$  denotes the small-scale i.i.d. Rayleigh fading coefficients.  $\mathbf{G} = \text{diag}(\mathbf{G}_{i,i})$  denotes the pathloss where  $\mathbf{G}_{i,i}$  is the pathloss from the BS to user  $i$ . Denote

$$\mathbf{H} = \mathbf{G} \mathbf{Q} \quad (5)$$

the channel matrix with the variance profile  $\mathbf{V}$ , i.e.  $\mathbb{E}[\|\mathbf{H}_{i,j}\|^2] = \mathbf{V}_{i,j}$ .

## III. BS POWER CONSUMPTION LOWER BOUND GIVEN SUM RATE REQUIREMENT

In this section, given the sum rate requirement in the cell, we consider at least how much power must be consumed at the BS to satisfy the requirement. Both the active number of RF chains and the BS transmit power are optimized to minimize the BS total power consumption. We begin by deriving a CFA of the downlink sum capacity upper bound, which in turn gives the  $P_{\text{BS}}$  lower bound. The large-scale antenna array assumption is made in our analysis, which means we assume  $N \ll M_{\text{tot}}$ , i.e. the number of active users is much less than the total number of available BS antennas. However only  $M$  out of  $M_{\text{tot}}$  antennas are active in consideration of the BS power consumption.

### A. Sum Capacity Upper Bound

The ergodic sum rate is upper bounded by the downlink broadcast channel ergodic sum capacity, which is given by [3] and [8]

$$R_{ub} = \mathbb{E}[\max_{\mathbf{D}} \log \det(\mathbf{H}^\dagger \mathbf{D} \mathbf{H} + \mathbf{I})] \quad (6)$$

*s.t.*  $\text{Tr}[\mathbf{D}] \leq P_t / \sigma_n^2,$

where  $\mathbf{D}$  is the diagonal dual uplink transmit covariance matrix,  $(\cdot)^\dagger$  denotes the conjugate transpose. It is well known that when SNR is high, the equal power allocation is the optimal solution to (6), i.e.

$$\mathbf{D}^* = \frac{P_t}{\sigma_n^2 N} \mathbf{I}. \quad (7)$$

We will focus on the high-SNR regime hereinafter. However, it is shown in the numerical results that our analysis gives a close approximation of the total BS power consumption under a

wide range of SNR since the static power (the part proportional to  $P_s$ ) dominates the total power consumption in the low-SNR regime.

*Theorem 1:* The capacity

$$C(P_t, M) = \mathbb{E} \log \det(\mathbf{I}_N + \frac{P_t}{L \sigma_n^2} \mathbf{H} \mathbf{H}^\dagger), \quad (8)$$

where  $\mathbf{H} \in \mathcal{C}^{N \times M}$  and the corresponding variance profile  $\mathbf{V}$  has identical columns, i.e.

$$\mathbf{V}_{1,j} = \mathbf{V}_{2,j} = \dots = \mathbf{V}_{N,j}, \forall 1 \leq j \leq M, \quad (9)$$

admits the closed-form deterministic approximation

$$C^*(P_t, M) \approx M \log\left(1 + \frac{1}{M} \sum_{i=1}^N \frac{\mathbf{V}_{i,1}}{\mathbf{V}_{i,1} \beta - z}\right) - \sum_{i=1}^N \log\left(\frac{-z}{\mathbf{V}_{i,1} \beta - z}\right) - \frac{\log(e) \sum_{i=1}^N \frac{\mathbf{V}_{i,1}}{\mathbf{V}_{i,1} \beta - z}}{1 + \frac{1}{M} \sum_{i=1}^N \frac{\mathbf{V}_{i,1}}{\mathbf{V}_{i,1} \beta - z}}, \quad (10)$$

where

$$\beta = \frac{z}{2\bar{V}} + \frac{M-N}{2M} + \sqrt{\left(\frac{z}{2\bar{V}} + \frac{M-N}{2M}\right)^2 - \frac{z}{\bar{V}}}, \quad (11)$$

$\bar{V} = \frac{1}{N} \sum_{i=1}^N \mathbf{V}_{i,1}$ , and  $z = -\frac{L \sigma_n^2}{M P_t}$  in the large system limit, i.e.  $M$  and  $N$  satisfy  $N \rightarrow \infty$  and

$$0 < \liminf_{N \rightarrow \infty} \frac{M}{N} \leq \limsup_{N \rightarrow \infty} \frac{M}{N} < \infty. \quad (12)$$

*Proof:* See Appendix A. ■

Note that the condition in (9) is interpreted as the pathlosses from one user to different antennas are identical, i.e. co-located antennas. In the high-SNR regime, the capacity expression in Theorem 1 can be further simplified by the following corollary.

*Corollary 1:* In the high-SNR regime, the capacity expression in (10) for  $M \geq N$  is given in (3) at the top of the page, where  $\tilde{V}$  denotes the geometric mean of the channel variance profile, i.e.

$$\tilde{V} = \left(\prod_{i=1}^N \mathbf{V}_{i,1}\right)^{\frac{1}{N}}. \quad (13)$$

The equation for  $M < N$  is cumbersome without any insight, thus it is not shown.

*Proof:* The key notion is that when SNR is high,  $z/\bar{V}$  is the reciprocal of the receive SNR, thus approaching zero. ■

*Corollary 2:* The capacity  $C^*$  in (10) with given user locations is no more than the capacity as if all the users are with the mean distance in the high-SNR regime, i.e.

$$C^*(P_t, M, \mathbf{V}_{1,1}, \mathbf{V}_{2,1}, \dots, \mathbf{V}_{N,1}) \leq C^*(P_t, M, \bar{V}). \quad (14)$$

*Proof:* See Appendix B.  $\blacksquare$

*Remark:* This corollary reveals that the user-location dispersion actually reduces the sum capacity when SNR is high, although some users may have better channels through which they can achieve high throughput. Basically this is because when SNR is high, it is more important to fully explore the multiplexing gain of the parallel channels than to have a few better channels to transmit, which is the optimal strategy in the low-SNR regime.

In the high-SNR regime, the sum capacity in (6) and (7) equals (10), thus Theorem 1 is used as a CFA of the sum capacity.

### B. The Optimal $P_t$ and $M$

Now we are in the place to analyze the optimal BS transmit power  $P_t^*$  and active antennas  $M^*$  in the sense of minimizing the total BS power consumption as in (2) leveraging the CFA we obtain in Theorem 1.

*Theorem 2:* When

$$R_{\text{req}} > N \log\left(\frac{P_s \tilde{V}}{\Delta_P \sigma_n^2 e \varphi(N)}\right), \quad (15)$$

where  $\varphi(N) = 1 - \frac{N}{(N+1)\frac{N+1}{N}}$ , the optimal RF chain circuit power consumption and the transmit power consumption satisfy

$$\log_e\left(\frac{M^*}{M^* - N}\right) = \frac{NP_s}{\Delta_P P_t^*}. \quad (16)$$

Specifically, in the high-spectrum-efficiency (HSE) regime,

$$\lim_{R_{\text{req}} \rightarrow \infty} M^* \rightarrow \infty \quad \text{and} \quad \lim_{R_{\text{req}} \rightarrow \infty} \frac{M^* P_s}{\Delta_P P_t^*} = 1, \quad (17)$$

i.e. the optimal RF chain circuit power consumption equals the transmit power consumption.  $M^*$  and the minimum total BS power consumption are

$$M^* = \sqrt{\frac{\Delta_P N \sigma_n^2 e^{\frac{R_{\text{req}}}{2N}}}{\tilde{V} P_s}}, \quad (18)$$

$$P_{\text{BS}}^{\text{min}} = 2 \sqrt{\frac{\Delta_P P_s N \sigma_n^2 e^{\frac{R_{\text{req}}}{2N}}}{\tilde{V}}}, \quad (19)$$

respectively in the HSE regime.

*Proof:* See Appendix C.  $\blacksquare$

*Remark 1:* The condition in (17) indicates the equal power allocation between transmit power and RF chain static power is optimal in the HSE regime. This condition has a simple explanation. Observing in the HSE regime, both  $M$  and  $P_t$  have the same effect on the sum capacity

$$C^*(P_t, M, \bar{V}) \approx N \log\left(\frac{M P_t}{N \sigma_n^2} \bar{V}\right). \quad (20)$$

Applying the Cauchy-Schwarz inequality based on the total power constraint, we can get (17) immediately.

*Remark 2:* Another important conclusion is that if the user-normalized sum rate requirement is fixed, i.e.  $\frac{R_{\text{req}}}{N} = \text{const.}$ , both  $M^*$  and  $P_{\text{BS}}^{\text{min}}$  are proportional to  $\sqrt{N}$ . This result gives us the answer to the problem what is the power-optimum

TABLE I  
SYSTEM PARAMETERS

Link related parameters	
Carrier frequency $f_c$	2.4 GHz
Bandwidth	5 MHz
Thermal noise	-174 dBm/Hz
Pathloss model	Pathloss = $36.7 \log_{10}(d) + 22.7$ $+ 26 \log_{10}(f_c)$
BS power consumption	
$P_s$	50W
$\Delta_P$	3.9

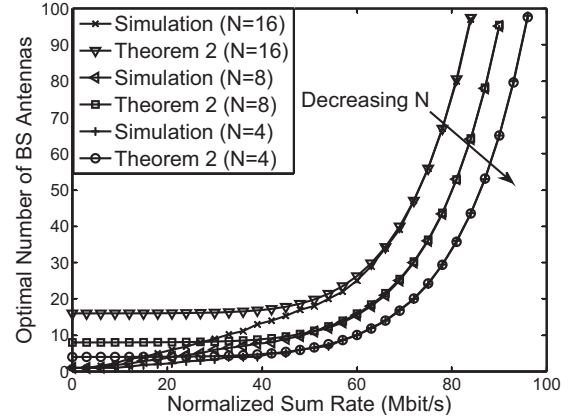


Fig. 1. The optimal number of active BS antennas versus the normalized sum rate.

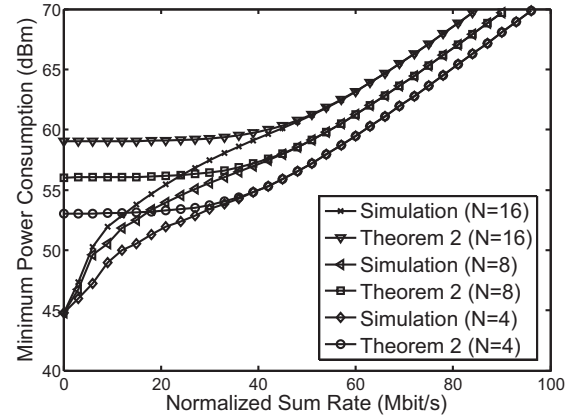


Fig. 2. The minimum BS power consumption versus the normalized sum rate.

number of active BS antennas for a specific number of users. Intuitively, the adding of one user brings one more degree-of-freedom (DoF), compromising the rate requirement increase brought by the user. Nevertheless, the loss in the power gain ( $M/N$ ) affects the power consumption, and it turns out to be of the form  $\sqrt{N}$ .

## IV. NUMERICAL RESULTS

The typical values of related parameters are shown in Table I. The users are generated by a uniformly distributed function

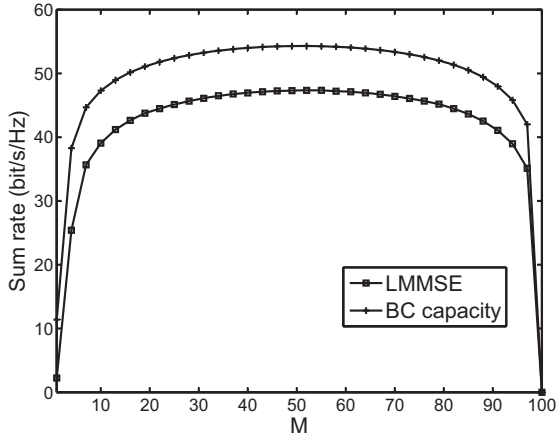


Fig. 3. The sum rate of LMMSE precoder and BC capacity versus the number of active BS antennas. The total BS power consumption is 3000W,  $P_s = 30W$ .

from the distance of 50m to 1000m. Fig. 1 and Fig. 2 show the optimal number of active antennas and minimum BS power consumption, computed by both simulations and Theorem 2 respectively. The  $M^*$  and  $P_{BS}^{\min}$  from Theorem 2 are shown to be close to the simulation results in the HSE regime.

In Fig. 3, the sum rate achieved by a practical linear minimum-mean-square-error (LMMSE) precoder is shown versus the number of active BS antennas  $M$ . The total power of the BS is fixed to 3000W, the antenna static power consumption  $P_s$  is 30W, and users are uniformly distributed from the distance of 50m to 1000m. The sum capacity of Theorem 1 is also shown. Since  $R(P_{BS})$  is monotonically increasing with  $P_{BS}$ , i.e. using more BS power can always improve the sum rate, the power-minimum problem in (2) is equivalent with the rate-maximum problem

$$\begin{aligned} & \text{Maximize} && R(P_{BS}), \\ & \text{s.t.} && MP_s + \Delta_P P_t \leq P_{BS}. \end{aligned} \quad (21)$$

Therefore, the rate-maximum number of active BS antenna is also the power-minimum one. Note that based on Theorem 2, under these parameters, the power-minimum number of active BS antennas is 50. This is evident of both the sum capacity and the LMMSE precoder shown in Fig. 3. The relationship between optimal static antenna power and transmit power in Theorem 2 is derived using the sum capacity of BC. Nonetheless, it is also applicable to practical linear precoders according to Fig. 3.

## V. CONCLUSION

This paper presents an theoretical analysis of the minimum power consumption of a BS with co-located large-scale antenna array, given the user sum rate requirement. A CFA of the sum capacity upper bound of BC with NID channel matrix is derived. The CFAs of the minimum BS transmit power, along with the optimal number of active antennas and optimal transmit power are derived when the user rate requirement is high. It is proved that to minimize the total BS

power consumption, the optimal power consumption of active antennas static power equals the power consumption related to the transmit power in the HSE regime. This property applies to some practical linear precoders such as LMMSE precoders as well.

Future work should consider the distributed antenna system (DAS), which has the potential to further reduce the power consumption of BSs by shortening the distance from BS antennas to users. However, the analysis for DAS is more challenging and intriguing since the homogeneity of the BS-antenna channel conditions no longer exists.

## APPENDIX A PROOF OF THEOREM 1

*Proof:* Based on [9, Theorem 2.3], the capacity in (8) admits the following deterministic equivalent expression in the large system limit when  $M$  and  $N$  satisfy (12):

$$\begin{aligned} C^*(P_t, M) &= \sum_{j=1}^M \log\left(1 + \frac{1}{M} \text{Tr}(\mathbf{D}_j \mathbf{T}_P)\right) \\ &\quad - \log \det\left(\frac{L\sigma_n^2}{MP_t} \mathbf{T}_P\right) \\ &\quad - \sum_{j=1}^M \frac{\frac{1}{M} \text{Tr}(\mathbf{D}_j \mathbf{T}_P)}{1 + \frac{1}{M} \text{Tr}(\mathbf{D}_j \mathbf{T}_P)} \log(e), \end{aligned} \quad (22)$$

where

$$\mathbf{D}_j = \text{diag}(\mathbf{V}_{1,j}, \dots, \mathbf{V}_{N,j}), \quad j = 1, \dots, M, \quad (23)$$

and  $\mathbf{T}_P = \mathbf{T}\left(-\frac{L\sigma_n^2}{MP_t}\right)$  with  $\mathbf{T}(z) = \text{diag}(t_1(z), \dots, t_N(z))$  defined by the following implicit equation

$$\mathbf{T}(z) = \left(\frac{1}{M} \sum_{j=1}^M \frac{\mathbf{D}_j}{1 + \frac{1}{M} \text{Tr}(\mathbf{D}_j \mathbf{T}(z))} - z \mathbf{I}_N\right)^{-1}. \quad (24)$$

Under the condition of (9),  $\mathbf{D}_j$  is irrelevant with  $j$ , thus (24) is rewritten as

$$\frac{1}{t_i(z)} = \frac{\mathbf{V}_{i,1}}{1 + \frac{1}{M} \sum_{j=1}^N \mathbf{V}_{j,1} t_j(z)} - z, \quad \forall 1 \leq i \leq N. \quad (25)$$

Define

$$\beta = \frac{1}{1 + \frac{1}{M} \sum_{j=1}^N \mathbf{V}_{j,1} t_j(z)}. \quad (26)$$

Plugging (26) into (25), we obtain

$$t_i = \frac{1}{\mathbf{V}_{i,1} \beta - z}. \quad (27)$$

Applying (27) into (26), then

$$\beta + \frac{1}{M} \sum_{j=1}^N \frac{\beta \mathbf{V}_{j,1}}{\beta \mathbf{V}_{j,1} - z} - 1 = 0 \quad (28)$$

Leveraging the approximation

$$\frac{1}{N} \sum_{j=1}^N \frac{\beta \mathbf{V}_{j,1}}{\beta \mathbf{V}_{j,1} - z} \approx \frac{\beta \bar{\mathbf{V}}}{\beta \bar{\mathbf{V}} - z}, \quad (29)$$

where  $\bar{V} = \frac{1}{N} \sum_{i=1}^N \mathbf{V}_{i,1}$ , we obtain the following equation

$$M\bar{V}\beta^2 + ((N-M)\bar{V} - Mz)\beta + Mz = 0. \quad (30)$$

Solve  $\beta$  based on (30), we can get (11). Apply this result to (27), then (10) follows, which concludes the proof. ■

#### APPENDIX B PROOF OF COROLLARY 2

*Proof:* Define

$$y = \sum_{i=1}^N \frac{\mathbf{V}_{i,1}}{\mathbf{V}_{i,1}\beta - z} > 0. \quad (31)$$

Based on the Jensen's equality and the fact that  $f(x) = -\log(\frac{-z}{\beta x - z})$  is concave in  $x$ , when  $\beta > 0$  and  $z < 0$ , we have

$$C^*(P_t, M, \mathbf{V}_{1,1}, \mathbf{V}_{2,1}, \dots, \mathbf{V}_{N,1}) \leq M \log\left(1 + \frac{1}{M}y\right) - \frac{y}{1+y} - \sum_{i=1}^N \log\left(\frac{-z}{\bar{V}\beta - z}\right). \quad (32)$$

Denote the right-hand-side of the above inequality  $\bar{C}^*(y, \bar{V})$ . Since  $g(x) = \frac{x}{x\beta - z}$  is convex in  $x$ , when  $\beta > 0$  and  $z < 0$ , and applying the Jensen's equality again, we obtain

$$y \leq N \frac{\bar{V}}{\bar{V}\beta - z}. \quad (33)$$

Along with the fact

$$\frac{\partial \bar{C}^*(y, \bar{V})}{\partial y} = \frac{\frac{1}{M}y}{1 + \frac{1}{M}y} > 0, \quad (34)$$

we have

$$C^*(P_t, M, V_{1,1}, V_{2,1}, \dots, V_{N,1}) \leq \bar{C}^*(y) \leq \bar{C}^*\left(N \frac{\bar{V}}{\bar{V}\beta - z}\right), \quad (35)$$

which concludes the proof. ■

#### APPENDIX C PROOF OF THEOREM 2

*Proof:* Define  $P_{BS}(i)$  the BS power consumption with optimal transmit power and  $i$  active antennas. Then if

$$P_{BS}(N+1) < P_{BS}(N), \quad (36)$$

along with the fact that the sum capacity in (10) is concave in  $M$  and  $P_t$  (verified by calculating the Hessian matrix), we can obtain  $M^* > N$ . Thus we can use the expression in (3) to compute  $M^*$ . The condition in (36) admits (15) after some manipulations of the equation. And we can get (16) leveraging the Lagrange method. Define the Lagrange function

$$\begin{aligned} L(P_t, M, \lambda) = & M \log\left(\frac{M}{M-N}\right) \\ & + N \log\left(\frac{M-N}{N} \frac{P_t \bar{V}}{\sigma_n^2}\right) - N \log(e) \\ & - \lambda(MP_s - \Delta_P P_t - P_{BS}). \end{aligned} \quad (37)$$

And by

$$\frac{\partial L(P_t, M, \lambda)}{\partial P_t} = \frac{\partial L(P_t, M, \lambda)}{\partial M} = 0, \quad (38)$$

and the fact that  $\lambda > 0$  ( $R_{\text{req}}$  is monotonically increasing with  $P_{BS}$ , thus the total power constraint is satisfied with equality), we can obtain (16). Then, based on (16),

$$\frac{NP_s}{\Delta_P P_t^*} = \log_e\left(1 + \frac{N}{M^* - N}\right) \leq \frac{N}{M^* - N}. \quad (39)$$

Applying (39) to (3), we obtain

$$\begin{aligned} R_{\text{req}} \leq & N \left( \log \frac{M^*}{M^* - N} + \log \frac{\Delta_P \bar{V} (P_t^*)^2}{P_s N \sigma_n^2} \right) \\ \leq & N \left( \log(N+1) + \log \frac{\Delta_P \bar{V} (P_t^*)^2}{P_s N \sigma_n^2} \right). \end{aligned} \quad (40)$$

The last inequality is because  $M^* \geq N+1$ . Therefore,

$$\lim_{R_{\text{req}} \rightarrow \infty} P_t^* \rightarrow \infty. \quad (41)$$

And along with (16), we obtain

$$\lim_{R_{\text{req}} \rightarrow \infty} M^* \rightarrow \infty. \quad (42)$$

Combining (42)(16) and the fact that

$$\lim_{M^* \rightarrow \infty} M^* \log_e\left(\frac{M^*}{M^* - N}\right) = N, \quad (43)$$

we can obtain

$$\lim_{R_{\text{req}} \rightarrow \infty} \frac{M^* P_s}{\Delta_P P_t^*} = 1. \quad (44)$$

And by plugging (44) into the capacity expression in (3), (18) and (19) follows, which concludes the proof. ■

#### REFERENCES

- [1] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590-3600, Nov. 2010.
- [2] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3936-3964, Sept. 2006.
- [3] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of gaussian mimo broadcast channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2658-2668, Oct. 2003.
- [4] X. Zhang, S. Zhou, Z. Niu, and X. Lin, "An Energy-Efficient User Scheduling Scheme for Multiuser MIMO Systems with RF Chain Sleeping," in Proc. *IEEE WCNC*, 2013.
- [5] Y. Pei, T. Pham, and Y. Liang, "How Many RF Chains are Optimal for Large-Scale MIMO Systems When Circuit Power is Considered?" in Proc. *IEEE Globecom*, 2012.
- [6] O. Arnold, F. Richter, G. Fettweis, and O. Blume, "Power consumption modeling of different base station types in heterogeneous cellular networks," *19th Future Network & Mobile Summit*, 2010.
- [7] J. Jose, A. Ashikhmin, T. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems," *IEEE Trans. Wireless Commun.*, no. 99, pp. 2640-2651, Aug. 2011.
- [8] W. Yu, "Uplink-downlink duality via minimax duality," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 361-374, Feb. 2006.
- [9] W. Hachem, P. Loubaton, and J. Najim, "A CLT for information-theoretic statistics of gram random matrices with a given variance profile," *Ann. Appl. Probab.*, vol. 18, no. 6, pp. 2071-2130, 2008.