Adaptive Bayesian Compressed Sensing Based Localization in Wireless Networks

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Abstract—This paper exploits the most recent developments in sparsity approximation and Compressed Sensing (CS) to efficiently perform localization in wireless networks. Based on the spatial sparsity of the mobile devices distribution, a Bayesian Compressed Sensing (BCS) scheme has been put forward to perform accurate localization. Location estimation is carried out at a network central unit (CU) in our scheme thus significantly alleviating the burden of mobile devices. Since the CU can observe correlated signals from different mobile devices, the proposed method utilizes the common structure of the received measurements in order to jointly estimate the locations precisely. Moreover, when the number of mobile devices changes, we increase or decrease the measurement number adaptively depending on “error bars” along with the immediate reconstruction processes. Simulation shows that the proposed method, i.e. Adaptive Multi-task BCS Localization (AMBL), results in a better accuracy in terms of mean localization error compared with traditional localization schemes.

Index Terms—Bayesian compressed sensing, localization, wireless networks, multi-task, localization error, error bars

I. INTRODUCTION

Accompanying with the explosive advancement of multimedia-rich networking data services, a special kind of applications has emerged in recent years, namely the so-called Location-based Services (LBSs) [1]. In order to realize LBSs, numerous techniques are available nowadays including GPS, GSM and WiFi. Though widely used, GPS is often energy-consumable and not suitable for central urban or indoor area with heavy building shadowing. Many indoor localization systems use received signal strength (RSS) measurements to estimate the locations of mobile devices. RSS values can be obtained by the mobile devices from Access Points (APs) in the area of interest. Especially, a pre-built radio environment map is utilized in the RSS-based localization methods in order to relieve from fluctuations of the RSS [2]. Among the others, k-nearest neighbor algorithm (KNN) is one of the most commonly applied map-based techniques, which estimates the location of the mobile device by calculating the centroid of k nearest neighbors that have smaller Euclidean distances to the radio environment map measurements.

Unfortunately, most of these localization techniques are often quite energy-intensive, making it difficult to run on mobile devices with limited processing power, short battery lifetime and small memory. On the other hand, the inherent sparsity of mobile devices in the physical space has motivated the use of the novel theory of Compressed Sensing (CS) [3][4] to get rid of computational burden. The sparsity is especially obvious when the physical space is divided into grids and each grid indicates a location area. Compressed Sensing provides a new framework that guarantees the accurate reconstruction of a sparse or compressible signal with far fewer measurements than that required by the Nyquist sampling theorem. The reconstruction is ensured by the fact that a small collection of linear projection of the original sparse signal contains enough information for signal recovery. Hence the sparse signal can be reconstructed exactly with high probability by solving an under-constraint problem with a plethora of methods [5].

A recent work [3] has shown that the theory of Compressed Sensing can be used to reconstruct the radio environment map based on the RSS measurements. The localization problem can be modeled as a sparsity problem since the mobile device is located at a specific point in the physical space at each time instant. However, the location estimation algorithm is still carried out on the mobile devices by averaging the RSS values. Relevant pre-phases are indispensable in order to exploit the CS approach, which is energy inefficient yet. Another work [6] also used CS to solve the localization problem. The relevant location estimation is formulated as an l1-minimization problem. Nonetheless, the intra- and inter-signal correlation structures present in the RSS measurements have not been taken into consideration and there are no criteria to determine if the current measurements are sufficient.

In this paper, we come up with an adaptive multi-task BCS localization (AMBL) scheme to dynamically determine the necessary number of measurements. By making the best of intra- and inter-signal correlation structures of the RSS measurements from different APs, we reduce the total amount of data required for accurate localization. Specifically, we improve the former multi-task Bayesian Compressed Sensing (MultiBCS) [7] algorithm in our new location estimation scheme. BCS algorithm provides a unique criterion on the accuracy of the reconstruction vector called “error bars” [8]. We can exploit the error bars to determine if the number of measurements is enough. Location estimation is carried out at a network central unit (CU) in our scheme thus significantly alleviating the burden of limited processing power, storage, and processing capabilities of the mobile devices.
The rest of this paper is structured as follows. In Section 2 we give an introduction to the fundamentals of CS and BCS techniques and describe the relevant MultiBCS framework. Section 3 depicts the system model and formulates our compressed sensing based localization approach. Numerical experiment details as well as simulation results that prove the effectiveness and accuracy of our method are presented and analyzed in Section 4. Finally, Section 5 concludes our work.

II. BAYESIAN COMPRESSED SENSING

The theory of CS states that if a real-world signal has a sparse representation in a certain basis then it can be recovered with significantly fewer samples or measurements than Nyquist-Shannon sampling theorem. Specifically, a signal \( x \in \mathbb{R}^N \) can be represented in terms of a \( N \times N \) transform basis \( \Psi \) such that

\[
x = \sum_{i=1}^{N} s_i \psi_i = \Psi s
\]

The signal \( x \) is called K-sparse if its sparse representation \( s \) has only \( K \) non-zero entries (\( \| s \|_0 = K \)), where \( K \ll N \).

Consider a linear projection process that \( y \) represents the linear projection of vector \( x \):

\[
y = \Phi x + E = \Phi \Psi s + E = \Theta s + E
\]

where \( \Phi \) is an \( M \times N \) measurement matrix, \( E = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N]^T \) represents the noisy environment effects with each entry \( \varepsilon_i \) being zero mean Gaussian random variable with variance \( \sigma^2 \).

Typically the reconstruction problem would be approximately treated as a convex problem as follows which can be solved by techniques like Orthogonal Matching Pursuit (OMP) [9] and Basis Pursuit (BP) [5]:

\[
\hat{s} = \arg \min_s \| s \|_1 \quad s.t. \quad \| y - \Phi s \|_2 \leq \varepsilon
\]

Compared with the typical CS algorithm, Bayesian Compressed Sensing [8] has better performance in reconstruction with noisy measurements \( y \) and has been verified to be closer to \( l_0 - \text{norm} \)[10]. Therefore, we can investigate the reconstruction process from the perspective of BCS framework. Besides achieving a better recovery performance, there are two other important issues we have considered: multitask BCS considering inter- and intra- signal correlations and a stopping criterion for determining when sufficient measurements have been performed.

Specifically let us assume there are \( P \) sets of CS measurements \( \{ y^i \}_{i=1}^{P} \) projected from \( P \) sets of original compressive signals \( \{ x^i \}_{i=1}^{P} \), denoted as follows:

\[
y^i = \Phi^i x^i + E^i = \Phi^i \Psi^i s^i + E^i
\]

where each \( x^i \in \mathbb{R}^N \) exploits a disparate random projection matrix \( \Phi^i \in \mathbb{R}^{M \times N} \) to derive \( y^i \in \mathbb{R}^M \), with every sparse vector in \( \{ s^i \}_{i=1}^{P} \) similar to each other. \( E^i \in \mathbb{R}^{M \times P} \) denotes Gaussian noise and \( P=1 \) represents the single task case. If these measurements are obtained from repeated experiment on similar scenes or the same type of tasks, then many of these \( P \) sets measurements are correlated, which is opportunistically fit for the case of RSS-gathering process in our scheme. MultiBCS algorithm can connect these recovery tasks together thus achieving efficient information-sharing between tasks.

Since \( E^i \) can be modeled as \( M^i \) i.i.d. zero mean Gaussian random variable with variance \( \sigma^2 \), we can express the likelihood function for \( s^i \) and \( \lambda_0 (\lambda_0 = 1/\sigma_0^2) \) deduced from (4) as

\[
p(y^i | s^i, \lambda_0) = \frac{1}{(2\pi)^{M/2} \sigma_0^2} \exp\left(-\frac{\lambda_0}{2} \| y^i - \Theta^i s^i \|_2^2 \right)
\]

The key idea of applying BCS to recover the compressive signal \( x \) is to establish a hierarchical prior to the sparse vector \( s \). In MultiBCS we just assign a common prior with common hyperparameters \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_N] \) to \( s^i \) as depicted in (6), by which we connect several recovery tasks together [7]:

\[
p(s^i | \lambda) = \prod_{j=1}^{N} N(s^i_j | 0, \lambda_j^{-1})
\]

where \( N(0, \lambda_j^{-1}) \) represents the Gaussian distribution with zero mean and variance \( \lambda_j^{-1} \). By the Bayes’ rule, using (5) and (6) the posterior for \( s \) can be described as:

\[
p(s^i | y^i, \lambda, \lambda_0) \propto p(y^i | s^i, \lambda_0)p(s^i | \lambda)\]

\[
= (2\pi)^{-M/2} \sigma_0^{-2} \exp\left(-\frac{\lambda_0}{2} \| s^i - \mu^i \|_2^2 \right) \quad \text{(7)}
\]

with mean and covariance given by

\[
\mu^i = \lambda_0 \Sigma^i \Theta^i \quad \Sigma^i = (\lambda_0 \Theta^T \Theta + \Lambda)^{-1} \quad \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)
\]

Subsequently, we have to estimate the hyperparameters \( \lambda \) and \( \lambda_0 \) in order to update the mean value \( \mu^i \) alternately. The Bayesian estimates for \( \lambda \) and \( \lambda_0 \) can be derived by maximizing the logarithm marginal likelihood

\[
L(\lambda, \lambda_0) = \sum_{i=1}^{P} \log p(y^i | \lambda, \lambda_0)
\]

\[
= \sum_{i=1}^{P} \log \int p(y^i | s^i, \lambda_0)p(s^i | \lambda)ds^i
\]

\[
= -\frac{M}{2} \sum_{i=1}^{P} \left[ M^i \log 2\pi + \log | C^i | + s^i(T(C^i)^{-1}s^i) \right]
\]

with

\[
C^i = \lambda_0^{-1} I + \Theta^i \Lambda^{-1} \Theta^iT
\]

where \( I \) is the unit matrix.

Afterwards, differentiate (9) with respect to \( \lambda \) and \( \lambda_0 \) respectively and then set the results to zero, yielding

\[
\lambda_j^{\text{new}} = \frac{P - \lambda_j \sum_{i=1}^{P} \Sigma^{i}_{j;j}}{\sum_{i=1}^{P} \mu_j^2}, j = 1, 2, \ldots, N
\]

\[
\lambda_0^{\text{new}} = \frac{\sum_{j=1}^{P} \left( M^j - N + \sum_{i=1}^{N} \lambda_j \Sigma_{j;j}^{i} \right)}{\sum_{i=1}^{P} \| y^i - \Theta^i \mu^i \|_2^2}
\]
Note that $\lambda^{\text{new}}$ and $\lambda_0^{\text{new}}$ are functions of $\mu^i$ and $\Sigma^i$, while $\mu^i$ and $\Sigma^i$ are pertinent to $\lambda$ and $\lambda_0$ shown in (8). So we can iterate between (8), (11) and (12) until convergence appears. Eventually the recovery of $s^i$ can be represented in terms of its mean $\mu^i$.

Furthermore the root-mean-squares of diagonal elements of the covariance matrix $\Sigma^i$ in (8) provide “error bars” (standard deviation) on the accuracy of the recovery of $s^i$. The following process indicates that “error bars” can easily be used to determine if the number of measurements is enough:

1. Set initial measurement number, $M^i = M_0^i$.
2. Run MultiBCS to reconstruct the compressive signals $\{s^i\}_{i=1}^P$ with $M^i_0$ measurements;
3. If the average “error bar” surpasses the predefined $E_0$, then increase $M^i + M^i = 1$;
   If the average “error bar” does not meet the predefined $E_0$, then decrease $M^i = M^i - 1$;
4. Return to Step 2 and update $M^i$.

III. COMPRESSED LOCATION SENSING

A. System Model

We consider a localization scenario as illustrated in Fig.1 which is divided into a set of $D$ non-overlapping grids of size $x_g \times y_g$. The area of interest is covered by $P$ access points (APs) and $K$ mobile devices (MDs) equipped with WLAN adapters. The locations of the MDs can be estimated by comparing the current RSS readings to a pre-set radio strength map of this area by the network central unit (CU). The radio strength map is a table of measured RSS readings of a similar device at every grid of the area.

Although a number of methods can be utilized to carry out the comparison between the RSS readings and the radio map, we choose Bayesian Compressed Sensing theory to find the best match, which is verified to be more accurate and more efficient. The inherent sparsity of the localization problem comes from the fact that a mobile device can be reasonably deemed to be located in exactly one of the grids. For simplicity, if the mobile device is located in the $i$-th grid, we can assume $s = [0, 0, 1, 0, 0, ..., 0, 0]^T \in R^D$ to be the location vector with its $i$-th element being equal to “1”. Then the proposed localization scheme reformulates the location estimation problem into a sparse signal recovery problem.

B. Compressed Sensing Based Localization

Because the number of mobile devices is very small compared to the number of grids, the property of sparseness is obeyed in this localization scenario ensuring the CS-based mechanism can be utilized. But none of the state-of-the-art CS-based localization algorithms provides any criterion to determine if the current measurements are sufficient. In this Section, we describe our compressed location sensing scheme which accurately locates the mobile devices in a noisy environment with an optimal number of measurements. The relevant localization processes are explained in detail as follows.

First of all, a set of RSS samples is collected at each grid from each AP. Let $\psi^i_j \in R^N$ denote the vector of receiving RSS measurements at grid $j$ from the $i$-th AP wherein $N$ indicates the sampling length. Then, these vectors are sent to a central unit via backhaul link, which constructs a single matrix $\Psi^i \in R^{N \times D}$ (the so-called radio map matrix) for the $i$-th AP by means of concatenating the corresponding $D$ vectors.

Common choices for transformation basis $\Psi$ are Discrete Fourier Transform (DFT) basis and Discrete Wavelet Transform (DWT) basis. Since sparseness of the location vector is obvious, we will prefer to choose the radio map matrix $\Psi^i$ to be the transformation basis. Therefore the actual received signals of MDs can be expressed as a linear combination of several columns of $\Psi^i$, which can also be explained as the multiplication product of the radio map matrix $\Psi^i$ and the $D \times 1$ location vector $s$.

Next a similar process to the previous step is followed. Mobile devices at the current unknown locations collect a train of RSS measurements from every AP. These measurements are sent to the CU afterwards. Considering the horrendous amount of measurements and the inherent sparsity property described above, CU exploits a measurement matrix $\Phi^i \in R^{M^i \times N}$ for the RSS sampling measurements $x^i \in R^N$ from the $i$-th AP to complete the incoherent projection. The measurement matrix $\Phi^i$ ought to be associated with each transform basis $\Psi^i$, where $M^i$ is the number of CS measurements for each AP. The overall RSS measurements for MDs associated with the $i$-th AP can be described as:

$$y^i = \Phi^i s^i + E^i = \Phi^i \Psi^i s^i + E^i = \Theta s^i + E^i$$  \hspace{1cm} (13)$$

where $P$ is the number of APs in this area and $E^i$ represents the Gaussian noise. $\Phi^i$ is a standard Gaussian matrix with its columns normalized to unit norm, thus guaranteeing the incoherence between $\Phi^i$ and $\Psi^i$. 

![Fig. 1. Localization scenario.](image-url)
Algorithm 1 AMBL

Initialization: \( M^i = M_0^i \)

1) CU collects RSS measurements from MDs;
2) Generate \( \Phi^i \in R^{M^i \times N} \) according to \( M^i \), \( i = 1, 2, ..., P \);
3) Obtain the measurements vector \( \{ y^i \}_{i=1}^P \) by projecting the RSS measurements with \( \Phi^i \);
4) Assign a common prior to the original sparse signal \( \{ s^i \}_{i=1}^P \);
5) Derive the mean and covariance of recovery location \( \hat{s} \) via (8) (11) (12) (13);
6) Increase \( M^i \) by 1 if the average error bar has surpassed the predefined \( E_b \) or decrease \( M^i \) by 1 if not and go back to 2). If the average error bar is near to the predefined \( E_b \), stop the iteration.

Outputs: the location vector \( \hat{s} \); the average of \( \mu^i \);
optimal number of measurement: \( M^i \)

End

IV. SIMULATION RESULTS

We conduct simulation for a ground plane of \( 20m \times 20m \). The area is discretized in grids of size \( 1.0m \times 1.0m \). The performances of the proposed Adaptive Multi-task BCS for Localization scheme are demonstrated and compared with other state-of-the-art methods to verify the accuracy and efficiency of our method. There are \( P=6 \) APs altogether in the additive white Gaussian noise-involved environment. The number of mobile devices is \( K \) which is relatively small. In our simulations we assume equal number of CS measurements for the \( i \)-th AP in AMBL, equivalently with the same \( M^i \) denoted as \( M = M^i \) in every task \( i \) for comparison simplicity and assume \( K=10 \) for Fig. 2, Fig. 3 and Fig. 4. We set the sample number of each RSS measurement from every AP to be 512, i.e. \( N=512 \). All the simulation results presented hereafter are an average of 100 simulation runs.

The accuracy of the estimated locations in our scheme is evaluated by Mean Localization Error (MLE), which is defined as follows with the recovered and original location vectors:

\[
MLE = \frac{\sum_{i=1}^{P} \left( \frac{\text{norm}(\hat{s}^i - s^i)}{\text{norm}(s^i)} \right)}{P}
\]

In Fig 2, we show the 2-dimension plot of the recovery and original positions of the MDs by means of the proposed AMBL scheme with the required error bar \( E_b = 0.10 \) and the initial number of measurements \( M_0 = 40 \). Zero-mean Gaussian noise is added to each of the \( M \) measurements which makes the SNR level fixed to 10dB.

Fig. 3 compares the localization error using BCS, MultiBCS and BP as a function of the number of measurements. Therein for MultiBCS we set the initial number of measurements of our AMBL scheme to be 20 and just increase \( M \) by 1 each time until \( M \) reaches 100, while the other settings remain the same as those of Fig. 2. As we expected, the localization error decreases by increasing the number of measurements. Moreover, the MultiBCS method excellently outperforms the other two approaches since it makes use of the common structure of original signals as prior information.

![Two dimension plot of the original position and recovery position](image-url)

Fig. 4 examines the localization error under different SNR level using BCS, AMBL and KNN. Each measurement is corrupted by Gaussian noise. The total number of measurements for BCS is fixed to 65. All the other parameters in AMBL are set identical to those of Fig. 2. The proposed AMBL method performs much better than the other methods, especially in the low SNR environment.

When the number of mobile devices increases, the optimal measurement number required in BCS and AMBL schemes also increases, which has been expressed in Fig. 5. In this situation, we have adjusted the original BCS method to adaptively choose the optimal measurement number as we have done for MultiBCS.

The number of measurements necessary for accurate reconstruction is adaptively changing in AMBL according to the environment with a fixed error bar \( (E_b = 0.10) \) and a fixed mean localization error \( (\text{MLE}=0.05) \). The other settings of Fig. 5 remain the same as those in Fig. 2. It is evident that our proposed AMBL method requires fewer measurements compared to the BCS method, especially when the number of...
MDs becomes large.

Fig. 6 illustrates the effect of stopping criteria $E_b$ in AMBL in the cases of 10 MDs and 20 MDs with $E_b$ changing from 0 to 0.2 and a fixed Mean Localization Error (MLE=0.05). The other settings remain the same as those in Fig. 2. We can observe that when $E_b$ is too small, it will require more measurements to achieve this small MLE. If performance is not in the first place, $E_b$ can be set larger to reduce the measurement costs. As it is known, there is a tradeoff between the accuracy and measurement number, thus we can choose an optimal $E_b$ to leverage this kind of tradeoff, which is a future work of our research.

V. CONCLUSION AND FUTURE WORK

In this paper, based on the theory of BCS, we proposed an adaptive multi-task BCS localization scheme to estimate the locations of MDs more accurately. Specifically we explored the intra- and inter- signal correlation structures of the RSS measurements and exploited the common structures to share information between different tasks. Moreover, we provided a criterion to determine if the current measurements are sufficient. Thus when the number of MDs changes, we can obtain the optimal measurement number adaptively with the help of error bars of the precedent reconstruction processes in our scheme. The proposed method has better localization accuracy with fewer measurements. The simulation results further verified the accuracy and efficiency of our scheme. Although how to choose an optimal $E_b$ to leverage localization accuracy and measurement number is beyond the focus of this paper, it’s an important issue in our future work.
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