Hard Combining Based Energy Efficient Spectrum Sensing in Cognitive Radio Network

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Abstract—In traditional hard combing cooperative detection strategy, the fusion center (FC) determines whether the spectrum of interest is idle or occupied by primary user (PU) after collecting the decisions from all secondary users (SUs) involved. This paper introduces two kinds of simple and computationally efficient spectrum sensing schemes with which the final decision at the FC can be reached faster. Specifically, we proposed hard combing based sequential detection and ordered transmission schemes under different signal-to-noise ratio (SNR) distribution assumptions. In these schemes, the FC performs hypothesis test each time after it receiving one decision from a SU until the final decision can be made reliably. The simulation results show the proposed techniques can significantly reduce the number of data required in identification of the spectrum hole while achieving the same error probability compared with traditional methods.

I. INTRODUCTION

Traditionally, licensed spectrum is allocated over relatively long time periods and is intended to be used by PU exclusively. This regulation results in substantial unused resources in frequency, time and space. Cognitive radio (CR) technique has been proposed as a potential paradigm to improve the effective utilization of limited spectrum resource [1], and it is an important component of the IEEE 802.22 standard being developed for wireless regional area networks. The Federal Communication Commission (FCC) has recently opened the white TV bands so that provides an opportunity to develop new wireless CR networks to utilize this VHF and UHF spectrum with superior propagation and building penetration. However, access to this newly opened spectrum also comes with some technical challenges such as the detection of signal under especial low SNR [2].

In the case of low SNR, relying on one SU to detect the presence of PU is highly unreliable. To improve the sensitivity of CR spectrum sensing even further, and to make it more robust against fading and the hidden terminal problem, cooperative sensing can be used. The concept of cooperative sensing is to use multiple sensors and combine their measurements into one common decision. This approach including soft combing and hard combing that are used most widely [3, 4]. Both of these strategies need a special terminal (separate node or one of the SUs) as the FC which decides whether the spectrum of interest is idle or occupied by PU based on the statistics received from SUs and then broadcast the final decision to them. For the soft combing strategy, SUs send their raw data such as log-likelihood ratios (LLRs) to the FC directly, for the later each SU makes its own individual decision, and transmits only a binary value to the FC.

Both of the combing strategies mentioned above can make the final decision only after the FC received statistic from all SUs, which will leads to large delay and energy consuming when a large number of SUs exist in the CR network. Several algorithm have been studied to reduce the number of transmission needed so as to reach the final decision more quickly [5]. In [6], sequential detection (SD) scheme is employed to offer the possibility of making highly reliable decision without unnecessary delay. [7] proposes an ordered transmission (OT) strategy that reduces the average number of sensor transmissions with no loss in error probability. However, to our best knowledge, previous works about SD and OT focused primarily on soft combing among multiply SUs [8], [9], which potentially requires a very large amount of data to be transmitted to the FC and so that causes a large delay [10]. Moreover, the data transmitted in soft combing will be badly deteriorated if the noise or interference is too large and deep fading exists, then the FC is prone to make a wrong decision in these cases. To overcome the drawbacks mentioned above, we proposed SD and OT based on hard combing in this paper. Hard combing scheme generally includes voting rule among which “OR” and “AND” rule are the two extreme cases that used commonly, it is simple but leads to even bigger error probability compared with noncooperative algorithm sometimes, therefore we apply optimum data fusion rule derived from Neyman-Pearson lemma, which always reduce the error probability [11]. In the proposed schemes, SUs transmit their binarized decisions to the FC sequentially, the FC performs a hypothesis test each time after it receives a binary value from one SU. Two thresholds were employed in the hypothesis test, when the sum is larger than the upper bound or smaller than the lower band, the FC makes the corresponding final decision, otherwise it requires further statistics. Specifically, when the SNRs for different SUs can be regarded equivalent, we employ the SD scheme, OT scheme is applied otherwise. In addition, to simplify the computation, we proposed the Gaussian approximation of the test statistic at the FC.

The remainder of this paper is organized as follows. In Section II, we describe the system model and formulated optimum data fusion rule. In Section III, we proposed hard combing based SD scheme when SNRs for all SUs are equivalent, and compare its performance with traditional method. For cases that SUs have different SNRs, OT is applied at hard combing.
and the advantage of it is illustrated by simulation in Section IV. Section V concludes this paper with a summary.

II. PROBLEM FORMULATION

A. Signal Model

Consider a CR network consists of $N$ SUs and one PU. Like in most related literature, to study the most important fundamental aspects of proposed signal detection schemes, we consider the additive Gaussian white noise (AWGN) channel in this paper. Furthermore, we assume the primary activity, whether on or off, does not change during the detection duration. The spectrum sensing can be modeled as a binary hypothesis as

$$H_0: \ x = n, $$
$$H_1: \ x = s + n,$$

where $s$ is a sequence of $M$ consecutively sampled signal symbols in a detection duration, $n$ is noise received by SUs, both of them are assumed to be independent and identically distributed (i.i.d) zero-mean circularly complex symmetric Gaussian with variance $\sigma^2_s$ and $\sigma^2_n$, respectively, that is $s \sim CN(0, \sigma^2_s)$ and $n \sim CN(0, \sigma^2_n)$. Each SU employs energy detector to make individual decision where they are combined according to certain lemma.

Fig.1 illustrates the procedures of hard combing spectrum sensing. Each SU makes its individual decision based on $T_e$ independently. Scalar $u_n \in \{0, 1\}$ denotes the decision of the $n^{th}$ SU, “0” declares empty of the channel and “1” means it is occupied by PU, it is obviously that $u_n$ obeys Bernoulli distribution showed in Table I.

<table>
<thead>
<tr>
<th>$u_n$</th>
<th>$L^n_{fa}$</th>
<th>$1 - L^n_{fa}$</th>
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<tbody>
<tr>
<td>$H_0$</td>
<td>$L^n_{fa}$</td>
<td>$1 - L^n_{fa}$</td>
</tr>
<tr>
<td>$H_1$</td>
<td>$1 - L^n_{md}$</td>
<td>$L^n_{md}$</td>
</tr>
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</table>

In the above table, $L^n_{fa}$ means $L_{fa}$ of the $n^{th}$ SU and $L^n_{md}$ has similar definition. The SUs send their decisions to the FC where they are combined according to certain lemma.

B. Optimal Neyman-Pearson Detector

We denote vector $u = [u_1, \ldots, u_N]$ as information the FC collects. Instead of traditional voting rule, here we employ Neyman-Pearson fusion lemma at the FC, with which we can obtain minimum missed detection probability subject to false alarm probability constraint. The Neyman-Pearson decision strategy in logarithmic form at the FC is

$$T = \ln \frac{P(u|H_1)}{P(u|H_0)} \geq \lambda.$$  

We denote the LLR at the $n^{th}$ SU as $T_n$, according to Table I, it can be derived as

$$T_n = \ln \frac{P(u_n|H_1)}{P(u_n|H_0)} = \begin{cases} \ln \frac{1 - L^n_{fa}}{L^n_{fa}}, & u_n = 1, \\ \ln \frac{L^n_{md}}{1 - L^n_{md}}, & u_n = 0. \end{cases}$$

Let $S_1$ and $S_0$ be the set of SUs that send “1” or “0” to the FC respectively, in this paper, we consider the case that test statistics from all SUs are independent, then (3) can be written as

$$T = \sum_{n=1}^{N} T_n = \sum_{n \in S_1} T_n + \sum_{n \in S_0} T_n = \sum_{n \in S_1} \ln \frac{1 - L^n_{fa}}{L^n_{fa}} + \sum_{n \in S_0} \ln \frac{F^n_{md}}{1 - F^n_{md}} \geq \lambda.$$  

The local LLRs are stored at the FC previously and the corresponding LLR will be added to $T$ when the FC receives “0” or “1” from a SU. The FC determines whether the spectrum is idle or busy when the sum is sufficient to make the final decision.

III. DETECTION UNDER EQUIVALENT SNRS

In this section, we assume the PU is far away from all SUs that locate closely, and then the distance between any two SUs is small compared with the distance from any SU to the PU, so that the received signal at each SU experiences almost identical path loss. Therefore, in the case of an AWGN environment, it is reasonable to assume that SNRs for different SUs are equivalent. In the following, we will compare the traditional and proposed detection strategy based on hard combing in terms of the number of SUs statistics (NSUS) required to finish the detection under specific requirements such as false alarm and missed detection probability at the FC, which we denote as $F_{fa}$ and $F_{md}$ respectively.

A. Fixed Sample Size Detection

The traditional hard combing detection is also called fixed sample size (FSS) detection since it involves all SUs statistics. For simplicity, we assume all SUs use the same false alarm probability, that is $L^n_{fa} = \ldots = L^n_{fa} = L_{fa}$, under the same SNR assumption, it also implying the missed detection probabilities for all SUs are equal to a value $L_{md}$. Denote $n_1$ as the number of SUs that report “1” to the FC, then $n_1$ obeys binomial distribution under either hypothesis, from Table I, we have

$$H_0: \ n_1 \sim B(L_{fa}, N),$$
$$H_1: \ n_1 \sim B(1 - L_{md}, N).$$

According to central limited theory (CLT), for large $N$, $n_1$ is approximately normally distributed as

$$H_0: \ n_1 \sim N(\mu_0, \sigma^2_0),$$
$$H_1: \ n_1 \sim N(\mu_1, \sigma^2_1),$$

where $\mu_0$ and $\mu_1$ are the expected numbers of successes, respectively, and $\sigma^2_0$ and $\sigma^2_1$ are the variances.
where $\mu_0 = NL_f a$, $\mu_1 = N(1-L_{md})$, $\sigma_0^2 = NL_f a(1-L_f a)$ and $\sigma_1^2 = NL_{md}(1-L_{md})$, with this approximation, we can find the NSUS for FSS scheme is

$$N = \left( \frac{a \sqrt{L_f a(1-L_f a) - b \sqrt{L_{md}(1-L_{md})}}}{1 - L_f a - L_{md}} \right)^2,$$

(8)

where $a = Q^{-1}(F_{fa})$ and $b = Q^{-1}(1-F_{md})$, please refer to Appendix A for the proof. The results are illustrated in Fig.2. We can observe from the figure that the NSUS needed in terms of $L_f a$ has a global minimum when given $F_{fa}$, $F_{md}$ and SNR. The optimal local false alarm probability $L_{fa}^*$ is given by

$$L_{fa}^* = \arg\min_{L_{fa}} N, \quad \forall L_{fa} \in (0,1)$$

(9)

it can be achieved when $\frac{dN}{dL_{fa}} = 0$. We can solve this equation numerically, details of which can be found in Appendix B, some solutions are showed in Fig.2 marked by asterisk. Once $L_{fa}^*$ has been computed, substitute it and the corresponding $L_{md}$ derived from (2) into (8), we get the minimum NSUS of FSS scheme, which we denote as $N_{mf}$.

**B. Sequential Detection**

In the above subsection, the number to finish the Neyman-Pearson decision is fixed, we can get the final decision only after all the statistics from SUs have been received by the FC. However, in our proposed SD scheme, the FC will collect statistics transmitted by SUs (i.e., “0” or “1”) sequentially. It performs a hypothesis test each time after receiving a statistic from a SU. The FC stops collecting further statistics and makes final decision when statistics gathered is sufficient for making a decision at a specified reliability level. Otherwise, it will acquire an additional statistic from another SU and repeat the above procedures until it terminates. According to [6][13], the decision strategy at the FC after receiving $N_s$ statistics from SUs is

$$\sum_{n=1}^{N_s} T_n \begin{cases} < \ln A, & \text{Declare } H_0 \\ \geq \ln B, & \text{Continue} \\ \geq \ln A, & \text{Declare } H_1 \end{cases}$$

(10)

Parameters in the above equation are

$$A = \frac{F_{md}}{1-F_{fa}}, \quad B = \frac{1-F_{md}}{F_{fa}}.$$

Note that the number needed to finish the sequential detection, i.e., $n=N_s$ will be a random variable, we deviate it in the following. The NSUS under either hypothesis in this case is

$$E[N_s|H_0] = \frac{(1-F_{fa})\ln A + F_{fa}\ln B}{E[T_n|H_0]},$$

(11)

$$E[N_s|H_1] = \frac{F_{md}\ln A + (1-F_{md})\ln B}{E[T_n|H_1]}.$$

(12)
A. Gaussian Approximation

CLT is applicable when all the local LLRs are i.i.d. When the SNRs of SUs are different, the LLRs in the SUs will be independent but not identical, so we can not use CLT directly, however, according to Lindeberg-Feller central limit theorem (LF-CLT) [14], if the difference between SNRs are not very large, which is normal in practice, then the test statistic in (5) can still be approximated as a Gaussian distribution. Without loss of generality, we assume SUs follow uniform distribution with $\delta$ uncertainty, that is, $SNR_\delta \sim U[SNR_0 - \delta/2, SNR_0 + \delta/2]$ where $SNR_0$ is the nominal signal-to-noise ratio. Then the LLR at the FC obeys following distribution,

$$
\begin{align}
H_0: & \quad T \sim \mathcal{N}(\mu_0, \sigma^2_0), \\
H_1: & \quad T \sim \mathcal{N}(\mu_1, \sigma^2_1).
\end{align}
$$

The expectation and variance of LLR at the FC under either hypothesis in this case can be calculated from Table I and (4) as follows,

$$
\begin{align}
\mu_0 &= \sum_{n=1}^{N} \left( L_{fa}^n \ln \frac{1-L_0}{L_{fa}} + (1-L_{fa}) \ln L_{md} \right) , \\
\mu_1 &= \sum_{n=1}^{N} \left( (1-L_{md}) \ln \frac{1-L_0}{L_{md}} + L_{md} \ln L_{fa} \right) , \\
\sigma^2_0 &= \sum_{n=1}^{N} L_{fa}^n (1-L_{fa}) \ln^2 L_{md}, \\
\sigma^2_1 &= \sum_{n=1}^{N} L_{md} (1-L_{md}) \ln^2 L_{fa}.
\end{align}
$$

Then the error probability in the FC can be derived as

$$
\begin{align}
F_{fa} &= P(T > \lambda | H_0) = Q(\frac{\lambda - \mu_0}{\sigma_0}), \\
F_{md} &= P(T < \lambda | H_1) = 1 - Q(\frac{\lambda - \mu_1}{\sigma_1}).
\end{align}
$$

Here we omit the mathematical proof of LF-CLT for it is trivial but some results are showed in Fig.4 for an intuitive notion of its correctness. It is clearly that the curve of simulation and theory are very close even when the noise uncertainty $\delta$ is as large as 4dB. With the Gaussian approximation, then we can numerically find the optimal local false alarm probabilities for all SUs and the corresponding threshold $\lambda$ that minimize $F_{md}$ under $F_{fa}$ constraint. These results are solved previously and then used in the following proposed transmission scheme.

B. Ordered Transmission

Here we propose an ordered transmission scheme (OT), details of which are as follows. Define

$$
T_{n}^m = \max(\ln \frac{1-L_0}{L_{fa}}, \ln \frac{L_{md}}{1-L_{fa}}),
$$

thereafter, SUs transmit their decisions, i.e., “0” or “1”, according to $T_{n}^m$ sequentially, the larger the earlier. Note that in contrast with the OT proposed in [7], where all SUs transfer their statistics after time in proportion to LLR, order here is ranged previously. All transmissions stop when the sum of LLRs corresponding to SUs that transmitted is larger than a threshold $t_U$ or smaller than a threshold $t_L$. Let $n_{UT}/T$ be the number of SUs that have not yet transmitted at a given time.

\[ E[T_n|H_0] = L_{fa} \ln \frac{1-L_{md}}{L_{fa}} + (1-L_{fa}) \ln L_{md} \]

\[ E[T_n|H_1] = (1-L_{md}) \ln \frac{1-L_{md}}{L_{fa}} + L_{md} \ln L_{fa} \]

Then the NSUS for the SD in this case is

$$
N_{ns} = \max\{E[N_s|H_0], E[N_s|H_1]\}. 
$$

C. Comparison of SD and FSS Schemes

Here we compare the theoretical and simulation NSUS of SD and FSS schemes, the results are illustrated in Fig.3. The samples is $M = 1000$, and is same in all following simulations. Note that for simulation, the SD scheme is truncated at a maximum NSUS $N_{max} = 3000$, the $N_{max}$ is chosen here such that it is at least three times larger than $N_{ns}$ in the SNR regime of interest. The truncating value ensures the SD terminates at last in any cases and has a negligible effect on the error probability at the same time. It can be seen from the figure that for both detection schemes, the simulation results are close to theoretical values. The proposed scheme uniformly gives significant saving in terms of NSUS compared with corresponding traditional one. Specifically, the results in the figure show that proposed SD needs only roughly half of NSUS the FSS scheme requires, which is an impressive saving to make a reliable decision at the FC.

IV. DETECTION UNDER DIFFERENT SNRS

In Section III, we assume the SNRs for different SUs are equal, however, they are different in some cases so that we can not employ SD scheme for it requires the local LLR obeys the same distribution. In order to reduce the NSUS in this case, we proposed an ordered information transmit method with which we only need a part of the SUs employed and not affect the detection performance at the same time. Still for the computation simplicity, we formulate the approximation of the test statistic at the FC at first.

[Fig. 4. The Gaussian approximation of $T$ for inequivalent SNRs, with SNR$_0$: -18dB, $\delta$: 4dB and $N$: 30.]

\[ E[T_n|H_0] = L_{fa} \ln \frac{1-L_{md}}{L_{fa}} + (1-L_{fa}) \ln L_{md} \]

\[ E[T_n|H_1] = (1-L_{md}) \ln \frac{1-L_{md}}{L_{fa}} + L_{md} \ln L_{fa} \]

Then the NSUS for the SD in this case is

$$
N_{ns} = \max\{E[N_s|H_0], E[N_s|H_1]\}. 
$$

\[ \delta = 0,1,2,4,6,8,10 \] 

\[ \lambda = 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1 \]
and let $T_{N-n_{UT}}^m$ denote the LLR corresponding to the last SU transmission prior to that same time. Define

$$t_U = \lambda + n_{UT} \left| T_{N-n_{UT}}^m \right|,$$  \hfill (22)  

$$t_L = \lambda - n_{UT} \left| T_{N-n_{UT}}^m \right|.$$  \hfill (23)

Then we have the following theorem,

**Theorem 1**: The OT scheme proposed involving a smaller average number of SU data transmissions compared with traditional optimum approach without any impact on the error probability at the FC.

**Proof**: Firstly we consider the case when the sum of the LLRs from the transmitting SUs is larger than upper bound $t_U$ in (22). Note that the transmissions have been ordered previously. Thus, if the LLR corresponding to the most recent transmission SU is $T_{N-n_{UT}}^m$, then the magnitude contribution from the sum of the LLRs related with the SUs that did not yet transmit will not larger than $n_{UT} \left| T_{N-n_{UT}}^m \right|$. Moreover, because the new threshold $t_U$ is the optimum threshold plus this extra safety margin, therefore once the sum of the LLRs related with SUs that have transmitted larger than $t_U$, this implies the sum of the LLRs from all SUs, including those who did not transmit, will have to be larger than $\lambda$. Thus, even without further transmission, it is reasonable to make the final decision at the FC. On the other hand, the event that the traditional scheme continues to transmit occurs with nonzero probability. So the average number of transmissions required by our scheme is less than that of traditional one. A similar savings of transmissions can be made when the sum of the LLRs related with transmitted SUs is smaller than $t_L$.  \hfill $\blacksquare$

### C. Simulation of Ordered Transmission

Denote $P_0 = P(H_0)$ and $P_1 = P(H_1) = 1 - P_0$ as the prior probability of the null and alternative hypothesis, that is, the possibility that spectrum of interest is idle or busy respectively. $N_{s0}$ and $N_{s1}$ presents the average number of transmissions saved (ANTS) under either hypothesis, then the average ANTS in total can be written as

$$N_s = P_0N_{s0} + P_1N_{s1}. \hfill (24)$$

Fig.5 plots $N_s$ as a function of the number of SUs under different prior probability of null hypothesis. We can observe from this figure that the ANTS is approximately in direct proportion to the number of SUs, and the prior probability has an impact on the ANTS but not very large. Generally speaking, $P_0$ is small, however, it can be seen from the figure that the proposed scheme can significantly reduce the number of transmissions even when $P_0$ is only 0.01. Fig.6 plots $N_s$ as a function of $\text{SNR}_0$ under different prior probability of hypothesis. It is obvious that the ANTS has a maximum value for each tested $P_0$. Although the prior probability has an impact, the OT scheme can still save about 10% transmissions when the $\text{SNR}_0$ is as low as -20dB.

### V. Conclusion

In this paper, we developed hard combing based SD and OT algorithms to reduce the time delay and energy consuming in spectrum sensing. Specifically, two thresholds are employed in the hypothesis test, the FC makes a hypothesis test each time after one statistic is received from the SU, when the sum larger than the upper bound or smaller than the lower bound, the FC makes the corresponding final decision, otherwise it requires further statistics. The extensive simulation results have shown that the proposed scheme can significantly reduce the number of data transmissions.

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### APPENDIX A

**THE NUMBER OF SU STATISTICS REQUIRED FOR FSS**

**Proof**: Note that

$$1 - L_{fa} - L_{md} = F_{\chi^2_{2M}} \left( \frac{2\lambda e}{\sigma^2} \right) - F_{\chi^2_{2M}} \left( \frac{2\lambda e}{\sigma^2 + \sigma^2_e} \right) > 0$$
for $F_{\chi_2^M}(\cdot)$ is a monotonic increasing function. Then we have

$$ (1 - L_{fa})(1 - L_{md}) > L_{fa}L_{md}, $$

(25)

thus from (5) and (7), the false alarm probability at the FC can be derived as follows,

$$ F_{fa} = P(T > \lambda|H_0) $$
$$ = P(n_1 + (N - n_1) \ln \frac{L_{md}}{1 - L_{fa}} > \lambda|H_0) $$
$$ = P(n_1 > \bar{n}|H_0) \quad ("\bar{n}" \text{because of (25)} ) $$
$$ = Q\left(\frac{\bar{n} - \mu_0}{\sigma_0}\right), $$

(26)

where

$$ \bar{n} = \frac{\lambda - N \ln \frac{L_{md}}{1 - L_{fa}}}{\ln \left(\frac{1 - L_{md}}{1 - L_{fa}}\right)}, $$

and $Q(\cdot)$ is the standard Gaussian complementary CDF. Similarly

$$ F_{md} = 1 - Q\left(\frac{\bar{n} - \mu_1}{\sigma_1}\right). $$

(27)

Then from (26) and (27), we obtain

$$ Q^{-1}(F_{fa})\sigma_0 + \mu_0 = Q^{-1}(1 - F_{md})\sigma_1 + \mu_1. $$

(28)

After some further computations, the number of SUs needed can be written as (8) in Section III.

**APPENDIX B**

**Optimal False Probability of Secondary Users**

*Proof:*

Take the derivation of $N$ at $L_{fa}$, we can obtain $L_{fa}^*$ is the solution of the following equation,

$$ 0 = \frac{a(1 - 2L_{fa})}{\sigma_0/\sqrt{N}} - \frac{b(1 - 2L_{md})}{\sigma_1/\sqrt{N}} \frac{dL_{md}}{dL_{fa}} (1 - L_{fa} - L_{md}) $$
$$ - \left(\frac{dL_{md}}{dL_{fa}} - 1\right). $$

(29)

where $\frac{dL_{md}}{dL_{fa}}$ will be derived in the following. From (2), we have

$$ F_{\chi_2^M}^{-1}(L_{fa}) = cF_{\chi_2^M}^{-1}(1 - L_{fa}). $$

Let

$$ y = F_{\chi_2^M}^{-1}(L_{md}), $$

then

$$ L_{md} = F_{\chi_2^M}(y) = \frac{1}{\Gamma(M)} \gamma(M, y/2). $$

Since

$$ \gamma(s, z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{z^{s+k}}{(s+k)} $$
$$ \frac{d\gamma(s, z)}{dz} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{z^{s+k-1}}{(s+k-1)} $$
$$ = z^{s-1} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} z^k = z^{s-1} e^z, $$

so

$$ \frac{dL_{md}}{dy} = \frac{1}{\Gamma(M)} \frac{d\gamma(M, y/2)}{dy} = \frac{1}{2\Gamma(M)} (y/2)^{M-1} e^{y/2}. $$

then

$$ \frac{dy}{dL_{fa}} = \frac{dL_{md}}{dL_{fa}} = \frac{1}{2\Gamma(M)} \frac{dL_{md}}{dL_{fa}} $$
$$ = \frac{1}{2\Gamma(M)} \frac{dL_{md}}{dL_{fa}} $$

(30)

In a similar way, if we let $y = cF_{\chi_2^M}^{-1}(1 - L_{fa})$, then we have

$$ \frac{dy}{dL_{fa}} = -\frac{c}{\sqrt{\pi}} \frac{dL_{md}}{dL_{fa}} $$

(31)

Compare (30) with (31), we have

$$ \frac{dL_{md}}{dL_{fa}} = -c \frac{L_{md}}{L_{fa}} c^{(1-c)/2}. $$

(32)

Substitute it into (29), then we can find the optimal local false alarm probability numerically.

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