**A High Energy Efficient Scheme with Selecting Sub-carriers Modulation in OFDM System**

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**Abstract**—With the tremendous growth in wireless networks market, unprecedented energy consumption and emissions of carbon dioxide ($\text{CO}_2$) is a growing concern for both operators and governments. OFDM technology has been adopted by many standards organizations due to its high spectrum efficiency, but induces poor energy efficiency of power amplifier (PA). A new scheme named Selecting Sub-Carriers Modulation (SSCM) to lower the input power of PA is given in this paper. SSCM divides the information bits into two parts of bits according to modulation mode: the first part of bits selects some sub-carriers of OFDM to transmit nothing, another part of bits is transmitted by the remainders. Sub-carriers transmitting nothing do not need energy, and fewer number of sub-carriers transmitting modulation symbols can improve PA’s efficiency because of the lower peak-to-average power ratio (PAPR). Analysis results show SSCM with different modulation modes can lower input power of PA with different proportion compared with traditional OFDM. Theoretical analysis and simulation results of the reliability of SSCM are given and match closely. In multi-path channel, SSCM with BPSK has the same performance as BPSK in traditional OFDM, while SSCM with QPSK performs worse than QPSK in traditional OFDM.

**Keywords**—Green Communication; Energy Efficiency; OFDM; PA; SSCM.

I. INTRODUCTION

The demand for wireless networks traffic has grown tremendously with communication technology development. The energy consumption of wireless networks causes not only the operators’ operational expenditure (OPEX) but also the social attention for carbon footprint. As showed in [1], Information and Communication Technology (ICT) represents about 2% of total carbon emissions, of which mobile networks account for around 0.2%. At present, the number of base stations (BSs) serving mobile users is increasing gradually, the energy consumption of each BS highly reaches to 25MWh per year and accounts for around 56% energy consumption of cellular networks. Radio of BS consumes about 80% of the energy requirement, and PA accounts for 50% of radio[1][2].

In order to decrease the emissions of $\text{CO}_2$, some organizations establish a series of communications standards or agreements to constrain energy consumption. Green communication is put forward to exploit new technology reducing energy consumption of the whole networks at all layers. In physical layer, re-designing RF heads is put forward to reduce power loss in feeder cables [3]; the Class J amplifier is used to improve the efficiency of PA in [2][4]. In link layer, a new multiple antenna transmission approach, called SM, has been applied to OFDM through the design of energy efficient MIMO schemes [5]; [6] gives a new concept called ultra wide time (UWT) and a new green algorithm for resource allocation, finds the energy minimization for wireless communications by convex optimization. In network layer, the authors of [7] discuss some methods about Cognitive Radio (CR) to cope with global warming and to enable global sustainable development; some strategies to control the mode of the BS are put forward, such as shutting down BS during low traffic [8]; the trade-off of energy efficiency-deployment efficiency, energy efficiency-spectrum efficiency, power-bandwidth and power-delay are presented in [9].

OFDM technology is adopted by many standards due to its high spectrum efficiency and resistance to inter-symbol interference (ISI). However, there exist two shortcomings: high PAPR and sensitivity to Doppler Frequency. Modern BSs are terribly inefficient because of their need for PA linearity and high PAPR. In order to ensure the quality of radio signals, PA has to operate well below saturation, resulting in poor power efficiency. Some researchers have presented methods to lower PAPR of OFDM by coding and decrease the power consumption to a certain extent in [10][11].

In this paper, in order to lower the energy consumption of PA in OFDM system, we propose a new scheme called SSCM. Keeping the same spectrum efficiency with OFDM, we try to reduce the number of sub-carriers bearing modulation symbols by selecting part of sub-carriers to modulate with information bits. On the one hand, the sub-carriers without bearing modulation symbols consumes no power; on the other hand, the less number of sub-carriers can lower the PAPR of input signal and improve the efficiency of PA, further to lower its energy consumption.

The rest of this paper is organized as follows: Section II presents the method selecting sub-carriers to modulate in detail, including the transmitter and receiver model. In Section III, we analyse the power consumption of PA with SSCM compared with traditional OFDM. And then, Section IV gives the theoretical analysis of the detection probability of sub-carriers bearing modulation symbols and the comprehensive reliability of SSCM-OFDM. We give the simulation results of the
performance of SSCM-OFDM compared with traditional OFDM in multi-path channel in Section V. Finally, Section VI concludes this paper.

II. SSCM-OFDM SYSTEM MODEL

This section will give the scheme of SSCM based on OFDM technology, including transmitter model and receiver model.

Considering an OFDM system, \( N_c \) denotes the number of sub-carriers of OFDM. Modulation mode is MPSK or MQAM, every symbol consists of \( m(=\log_2 M) \) bits. The length of OFDM symbol is \( T \) and \( \Delta f = 1/T \) denotes the sub-bandwidth. \( f_0 \) denotes center frequency of OFDM system.

A. Transmitter Model

The transmitter model is showed in Fig.1 and with it the transmitting process of SSCM-OFDM is illustrated as follows:

![Figure 1. Transmitter Model of SSCM-OFDM](image)

1) Transmitter divides the \( N_c \) sub-carriers of OFDM into \( N = N_c / M \) groups according to the size \( M \) of modulation constellation;
2) Matrix \( D \) of the size \( N \times Mm \) denotes the data to be transmitted in one OFDM symbol. Departing matrix \( D \) into matrix \( A \) and matrix \( B \), they will be used in step 3) and step 4) respectively.

\[
D = [A|B] = \begin{bmatrix}
  a_{1,1} & \cdots & a_{1,m} & b_{1,1} & \cdots & b_{1,(M-1)m} \\
  a_{2,1} & \cdots & a_{2,m} & b_{2,1} & \cdots & b_{2,(M-1)m} \\
  \vdots & & \vdots & \vdots & & \vdots \\
  a_{N,1} & \cdots & a_{N,m} & b_{N,1} & \cdots & b_{N,(M-1)m}
\end{bmatrix}
\] (1)

3) Selecting a sub-carrier \( e_k \) from the \( k \) th group sub-carriers as an empty carrier according to the bit in the \( k \) th row of matrix \( A \). The expression of \( e_k \) is showed in (2) and implemented by Calculator module in Fig.1. \( E \) denotes the vector \([e_1,e_2,\cdots,e_v]\).

\[
e_k = \sum_{i=1}^{m} 2^{i-1} a_{k,i} + 1
\] (2)

4) Modulating the \( k \) th row of matrix \( B \) into the vector \( X_k = [X_{k,1},\cdots,X_{k,v-1},0,X_{k,v+1},\cdots,X_{k,M}] \) \( (k=1,2,\cdots,N) \). As a result, besides the sub-carrier \( e_k \), the remainder \( M-1 \) sub-carriers transmit the \( M-1 \) symbols in the \( k \) th group.

5) Then \( x(t) (0 \leq t \leq T) \) of the \( k \) th group modulated sub-carriers can be expressed in (3). And through IFFT, the SSCM-OFDM symbol \( x(t) \) can be expressed in (4).

\[
x_k(t) = \sum_{i=1}^{M} X_{k,i} \exp\{j2\pi(f_0 + i\Delta f)t\}
\] (3)

\[
x(t) = \sum_{k=1}^{N} \sum_{i=1}^{M} X_{k,i} \exp\{j2\pi(f_0 + i\Delta f)t\}
\] (4)

From the steps above, the spectrum efficiency equals to the bits every sub-carrier of SSCM-OFDM can load, i.e.,

\[
\eta_{SSCM-OFDM} = N(\log_2 M + (M-1)\log_2 M) / N_c = m
\] (5)

And every sub-carrier of traditional OFDM can load \( \eta_{OFDM} \) bits,

\[
\eta_{OFDM} = \log_2 M = m
\] (6)

So SSCM-OFDM can reach the same spectrum efficiency as traditional OFDM.

B. Receiver Model

At the receiver of SSCM-OFDM, one group sub-carriers’ modulation symbols after being equalized in frequency domain can be expressed in (7).

\[
Y_k = X_k + N_k = \begin{bmatrix}
  X_{k,1} + N_{k,1} \\
  X_{k,2} + N_{k,2} \\
  \vdots \\
  X_{k,v} + N_{k,v} \\
  \vdots \\
  X_{k,M} + N_{k,M}
\end{bmatrix}
\] (7)

where \( X_k, N_k, Y_k \) denote the \( k \)th group transmitted data, noise and equalized received signals respectively in frequency domain, \( X_{k,v} \), \( N_{k,v} \), \( Y_{k,v} \) are their elements.

Assuming the power of \( X_{k,v} \) is normalized, the average power of \( N_{k,v} \) is \( \sigma^2 \), and the data \( X_{k,v} \) is independent to the noise \( N_{k,v} \). The average power of sub-carrier bearing information is higher than the sub-carrier with no information, i.e.,

\[
E[|Y_{k,v}|^2] = E[|X_{k,v}|^2 + |N_{k,v}|^2] = 1 + \sigma^2 > E[|N_{k,v}|^2] = \sigma^2
\] (8)

Therefore, the principle of detecting the sub-carrier with no information symbols is:

\[
e_k^* = \arg \min_j |Y_{k,v}|
\] (9)

![Figure 2. Receiver Model of the kth Group Sub-carriers](image)
remainder sub-carriers $\{1, 2, \ldots, e_i^2 - 1, e_i^2 + 1, \ldots, M\}$ are demodulated into the binary bits, then combining them to get all data information.

III. ANALYSIS OF ENERGY EFFICIENCY

We will discuss the energy efficiency of SSCM-OFDM compared with traditional OFDM in this section.

According to [12], the energy efficiency $\eta_{PA}$ of PA is showed in (10), where $k = 0.5$, $P_{in}$ and $P_{out ave}$ denote input power and average output power of PA respectively.

$$\eta_{PA} = P_{out ave} / P_{in} = k / \text{PAPR} \quad (10)$$

The PAPR of OFDM is defined as:

$$\text{PAPR} = \max_{0 \leq t \leq T} \left\{ \left| x(t) \right|^2 \right\} / E\left[ \left| x(t) \right|^2 \right] \quad (11)$$

Then the relationship between $P_{in}$ and $P_{out ave}$ of PA can be expressed as:

$$P_{in} = P_{out ave} \cdot \text{PAPR} / k = 2P_{out ave} \cdot \text{PAPR} \quad (12)$$

When the average output power of PA is constant, the formula between the average input power of PA and the average PAPR is

$$E[P_{in}] = E[2P_{out ave} \cdot \text{PAPR}] = 2P_{out ave} E[\text{PAPR}] \quad (13)$$

Equation (13) shows that the average energy consumption of PA is in proportion to $E[\text{PAPR}]$. To simplify the computation complexity, we use the PAPR value, which makes the Cumulative Distribution Function (CDF) equal to 0.5, instead of $E[\text{PAPR}]$. In [12] the CDF function of PAPR was given,

$$P_r(\text{PAPR} < \gamma) = \exp\left(-\sqrt{\pi \gamma / 3N_s e^{-\gamma}}\right) \quad (14)$$

The number of sub-carriers bearing modulation symbols in SSCM-OFDM is:

$$N_{SSCM} = N(2^n - 1) / 2^n \quad (15)$$

The ratio of $N_{SSCM}$ to the number of sub-carriers in traditional OFDM is:

$$k_{sscm} = N_{SSCM} / N_c = (2^n - 1) / 2^n \quad (16)$$

We can get the PAPR comparison between SSCM-OFDM and OFDM, just as showed in Table I, where $\gamma_{0.5, \text{SSCM}}$ or $\gamma_{0.5, \text{OFDM}}$ is the PAPR that makes CDF=0.5 in traditional OFDM or SSCM-OFDM respectively, $\Delta$ is the gap between them, $l_f$ is the linear value, i.e. $\Delta = -10 \log_{10} l_f$.

**Table I. PAPR of SSCM-OFDM and OFDM (CDF=0.5)**

<table>
<thead>
<tr>
<th>Modulation</th>
<th>BPSK</th>
<th>QPSK</th>
<th>16QAM</th>
<th>64QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{0.5, \text{OFDM}}$ [dB]</td>
<td>9.6</td>
<td>9.6</td>
<td>9.6</td>
<td>9.6</td>
</tr>
<tr>
<td>$\gamma_{0.5, \text{SSCM}}$ [dB]</td>
<td>9.235</td>
<td>9.452</td>
<td>9.567</td>
<td>9.592</td>
</tr>
<tr>
<td>$\Delta$ [dB]</td>
<td>0.365</td>
<td>0.148</td>
<td>0.033</td>
<td>0.008</td>
</tr>
<tr>
<td>$l_f$</td>
<td>1.0087</td>
<td>1.0347</td>
<td>1.0076</td>
<td>1.0018</td>
</tr>
</tbody>
</table>

From Table I, we can see the PAPR of SSCM-OFDM is lower than that of traditional OFDM when the modulation is BPSK or QPSK; however the range varies slightly when the modulation order increases, because the number of sub-carriers bearing symbols of SSCM-OFDM is close to the number of sub-carriers of traditional OFDM.

Apart from the high efficiency of PA, another main reason resulting in lower input power of PA is the sub-carriers bearing no symbol need no power. The ratio of input power of PA in SSCM-OFDM to the input power in traditional OFDM is defined as $l_f = (2^n - 1) / 2^n$, and $\Delta = -10 \log_{10} l_f$. So we can get the comprehensive ratio $\eta$ of the power consumed by PA in SSCM-OFDM to the power consumed by PA in traditional OFDM,

$$P_{out ave \cdot \text{SSCM}} = l_f P_{out ave \cdot \text{OFDM}}$$

$$\Delta_{\text{c}} = -10 \log_{10} \eta \quad (17)$$

$$E[P_{\text{SSCM}}] = 2P_{out ave \cdot \text{SSCM}} E[\text{PAPR}_{\text{SSCM}}] \quad (18)$$

$$\approx 2l_f P_{out ave \cdot \text{OFDM}} \gamma_{0.5, \text{SSCM}}$$

$$\approx 2l_f P_{out ave \cdot \text{OFDM}} \gamma_{0.5, \text{OFDM}} / l_f$$

$$\eta = P_{in \cdot \text{SSCM}} / P_{in \cdot \text{OFDM}} \approx (2^n - 1) / (2^n l_f)$$

$$\Delta_l = -10 \log_{10} \eta \quad (19)$$

The approximate power consumption of PA in SSCM-OFDM compared to traditional OFDM with different modulation modes are showed in Table II.

**Table II. Power Consumption of SSCM and OFDM**

<table>
<thead>
<tr>
<th>Modulation</th>
<th>BPSK</th>
<th>QPSK</th>
<th>16QAM</th>
<th>64QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_\gamma$ [dB]</td>
<td>0.365</td>
<td>0.148</td>
<td>0.033</td>
<td>0.008</td>
</tr>
<tr>
<td>$\Delta_n$ [dB]</td>
<td>3.01</td>
<td>1.249</td>
<td>0.28</td>
<td>0.068</td>
</tr>
<tr>
<td>$\eta$</td>
<td>3.375</td>
<td>1.397</td>
<td>0.313</td>
<td>0.076</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>45.97%</td>
<td>72.49%</td>
<td>93.05%</td>
<td>98.27%</td>
</tr>
</tbody>
</table>

Just as the analysis shows: SSCM-OFDM reduces the energy consumption by decreasing the number of sub-carriers bearing modulation symbols, meanwhile the efficiency of PA is improved as a result of the lower PAPR. Compared with traditional OFDM, SSCM-OFDM with BPSK or QPSK can sharply lower 54.03% or 27.51% of the PA's consumption respectively, however, 16QAM and 64QAM slightly lower 6.95% and 1.73% respectively.

Except for high energy efficiency, the sub-carriers bearing no symbol will not interference the other sub-carriers in the same SSCM-OFDM symbol; meanwhile, it will not interference the other users using the same sub-carriers. However, every coin has two sides, the reliability of SSCM-OFDM is slightly poorer than traditional OFDM, which will be analysed detailedly in the following section.

IV. RELIABILITY OF SSCM-OFDM

Through multi-path channel, the received signal will be equalized firstly. $N_{\gamma}$ in (7) can be saw as AWGN in order to simplify theoretical analysis, so we only discuss the theoretical reliability of SSCM-OFDM in AWGN channel. Meanwhile, SSCM-OFDM with 16QAM or 64QAM lowering the energy consumption is insignificant, so we only consider the theoretical values of SSCM-OFDM with BPSK and QPSK in this section.
A. Detection Probability of Sub-carrier Bearing Information

Considering the detection probability \( \Pr \{ e'_i = e'_j \} \) of sub-carrier bearing no symbol in a group, we can ignore the group sign \( k \). Assuming data is independent to the noise, the distribution function of \( |Y_i|^2 \) is showed in (21),

\[
|Y_i|^2 = \begin{cases} 
X_i + N_i^2 \sim \frac{1}{2\sigma} \exp \left( -\frac{x}{2\sigma} \right), x > |X|^2 \ (i \neq e); \\
N_i^2 \sim \frac{1}{2\sigma} \exp \left( -\frac{x}{2\sigma} \right), x > 0 \ (i = e).
\end{cases}
\]  

(21)

Assuming \( g_x = \min \{ |Y_i|^2, g_x \} \), then the distribution function of \( g_x \) is

\[
P \{ g_x < y \} = P \{ \min \{ |Y_i|^2, g_x \} < y \} \\
= 1 - P \{ \min \{ |Y_i|^2, g_x \} > y \} \\
= 1 - \prod_{i \neq e} \left( 1 - P \{ |Y_i|^2 > y \} \right) \\
= 1 - \exp \left\{ - (M - 1) (y - |X|^2) / 2\sigma^2 \right\}
\]

and the probability density function of \( g_x \) is

\[
f_{g_x}(y) = \frac{(M-1)}{2\sigma^2} \exp \left\{ - \frac{(M-1)(y-|X|^2)}{2\sigma^2} \right\}, y > |X|^2
\]  

(23)

Then the correct detection probability \( P_c \) of the sub-carrier bearing symbol is

\[
P_c = P \{ e'_i = e \} = P \{ g_x > g_x \} \\
= \int_{y=-\infty}^{\infty} \frac{(M-1)}{2\sigma^2} \exp \left\{ - \frac{(M-1)(y-|X|^2)}{2\sigma^2} \right\} dy \\
= \left[ - \frac{(M-1)}{2\sigma^2} e^{-\frac{|X|^2}{2\sigma^2}} \right]_{0}^{\infty}
\]

(24)

and the error detection probability \( P_{det} \) is

\[
P_{det} = 1 - P_c = \frac{(M-1)}{2\sigma^2} e^{-\frac{|X|^2}{2\sigma^2}} \xrightarrow{\sigma^2 \to 0} 0;
\]

(25)

B. Comprehensive Reliability of SSCM-OFDM

Assuming \( P_{e_{\text{MPSK}}} \) denotes the BER of traditional OFDM with MPSK in AWGN channel, \( P_{dem} \) denotes the BER of bits (contained in matrix B) bear in sub-carriers of SSCM-OFDM. The relation between them is

\[
P_{dem} = 1 - P_c (1 - P_{e_{\text{MPSK}}}) > 1 - (1 - P_{e_{\text{MPSK}}}) = P_{e_{\text{MPSK}}}
\]

(26)

So \( P_{dem} \) is poorer than \( P_{e_{\text{MPSK}}} \). But actually, about half of the bits bear in sub-carriers are correct when the detection of non-empty sub-carriers is error, so the approximate theoretical formula is

\[
P_{dem} \approx 1 - P_c (1 - P_{e_{\text{MPSK}}}) - P_{det} / 2
\]

(27)

Further, the comprehensive BER \( P_e \) of all the bits (contained in matrix D) of SSCM-OFDM symbol including the bits selecting sub-carriers (contained in matrix A) and the bits (contained in matrix B) bear in sub-carriers is

\[
P_e = \left[ (2^m - 1) P_{dem} + P_{det} \right] / 2^m
\]

(28)

\( P_e \) must be between the BER \( P_{det} \) and the BER \( P_{dem} \), i.e.,

\[
P_e \in \left[ \min \{ P_{det}, P_{dem} \}, \max \{ P_{det}, P_{dem} \} \right]
\]

(29)

The rest of this section gives the numerical results and theoretical results of \( P_e, P_{det}, P_{dem} \) in AWGN channel when the modulation modes are BPSK or QPSK. The main parameters of SSCM-OFDM are same as Table III. Here the theoretical \( P_{e_{\text{MPSK}}} \) of BPSK or QPSK in OFDM is approximated by the \( P_{e_{\text{MPSK}}} \) of BPSK or QPSK in AWGN channel, i.e.,

\[
P_{e_{\text{MPSK}}} = 2Q \left( \frac{E_s}{N_0} \sin \frac{\pi}{4} \right)
\]

(30)

where \( E_n = E_s \log_2 M \), \( E_s \) denotes the energy of every bit.
From Fig. 4 and Fig. 5, we can see the comprehensive BER $P_e$ of SSCM-OFDM is between $P_{det}$ and $P_{dem}$, $P_{dem}$ is less than $P_{det}$, and the theoretical results approximate the simulation results.

V. PERFORMANCE OF SSCM-OFDM VERSUS OFDM

One advantage of OFDM is high spectrum efficiency, another is resistance to ISI, and the actual channel is not the ideal AWGN channel. So it necessary to evaluate the performance of SSCM-OFDM with BPSK and QPSK compared with the traditional OFDM in multi-path channel.

### TABLE IV. PARAMETERS OF MULTI-PATH CHANNEL

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Path</td>
<td>12</td>
</tr>
<tr>
<td>Max Delay</td>
<td>88(Point)</td>
</tr>
<tr>
<td>Max Doppler Frequency</td>
<td>80Hz</td>
</tr>
<tr>
<td>Multi-path Power Spectrum</td>
<td>Exponent Power Spectrum</td>
</tr>
<tr>
<td>Channel Estimation</td>
<td>Ideal Channel Estimation</td>
</tr>
<tr>
<td>Equalization</td>
<td>LS</td>
</tr>
</tbody>
</table>

The main parameters about multi-path channel are listed in Table IV and the main parameters of OFDM are same as Table III. The simulation results are showed in Fig. 6. The results show that SSCM-OFDM with BPSK perform the same as BPSK in traditional OFDM, while SSCM-OFDM with QPSK performs worse than QPSK in traditional OFDM due to the effect of equalization in multi-path channel. The gap can be minimized by other more effective equalization algorithms.

VI. CONCLUSION

In this paper a new scheme called SSCM in multi-carriers system is developed in order to lower the input power of PA. The main idea is to reduce the number of sub-carriers of OFDM, it not only reduces the energy required by sub-carriers of OFDM, but also improves the efficiency of PA to save the input power. Given the transmitter and receiver model of SSCM-OFDM, our analysis shows SSCM-OFDM with BPSK and QPSK can lower 54.03% and 27.51% of PA consumption compared with traditional OFDM respectively. Simulation of the reliability of SSCM-OFDM manifests that numerical results approach theoretical results. SSCM-OFDM with BPSK has the same performance with BPSK in traditional OFDM, while SSCM-OFDM with QPSK performs worse than QPSK in traditional OFDM in multi-path channel.

REFERENCES